Price Twenty-Five Cents.

VOL. I. NO. IV.

THE

MATHEMATICAL MONTHLY.

JANUARY, 1859.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

CAMBRIDGE:
PUBLISHED BY JOHN BARTLETT.
LONDON:
TRÜBNER AND CO.
1859.

CONTENTS.

JANUARY, 1859.

	Page
Prize Problems for Students,	117
The Order of Mathematical Studies. By Rev. Thomas Hill,	118
Propositions Relating to the Cone and Sphere. By Prof. E. S. Snell,	121
Magic Square for the Year 1859. By J. Weissner,	122
Extraction of the Higher Roots of Numbers. By Prof. Dascom Greene,	123
Investigation of the Nature of the Curve of a Drawbridge. By William Watson,	125
On a Simplification in Computing Earthwork. By J. B. Henck,	127
Demonstration of a Theorem. By A. Cayley, F. R. S.,	130
Notes on the Hypocycloid. By Prof. O. Root,	133
Note on the Equation of the Projection of a Great Circle of the Globe on a Mercator Chart. By Prof. W. Chauvenet,	135
Notes on the Theory of Probabilities. By Simon Newcomb,	136
On the Motions of Fluids and Projectiles Relative to the Earth's Surface. By Prof. W. Ferrel,	140
Problem. By J. Foster Flagg,	148
Mathematical Monthly Notices,	149
Editorial Items,	150
Report of the Committee on the Solutions of the Prize Problems in the October Number of the Monthly,	152

TERMS.

A single Copy,	$3.00	per annum.
Two Copies to one Address,	5.00	"
Five Copies " "	11.00	"
Ten Copies " "	20.00	"

Payable invariably in advance.

JOHN BARTLETT, *Publisher.*

Entered according to Act of Congress in the year 1859, by
J. D. RUNKLE,
In the Clerk's Office of the District Court of the District of Massachusetts.

CAMBRIDGE:
ALLEN AND FARNHAM, ELECTROTYPERS AND PRINTERS.

A COURSE OF NATURAL PHILOSOPHY.

BY W. H. C. BARTLETT,

PROFESSOR OF NATURAL AND EXPERIMENTAL PHILOSOPHY IN THE MILITARY ACADEMY OF THE UNITED STATES.

DESIGNED FOR

COLLEGES AND ACADEMIES.

PUBLISHED BY A. S. BARNES & CO., 51 & 53 JOHN STREET, NEW YORK.

1. Elements of Synthetical Mechanics.	**Price $3.00**	**3. Spherical Astronomy.**	**Price $3.00**
2. Elements of Analytical Mechanics.	**" 4.00**	**4. Acoustics and Optics.**	**" 2.00**

TO PRESIDENTS, PROFESSORS, AND TEACHERS.

The undersigned respectfully invite your attention to the above Text-Books, and to the following testimonials in their behalf: —

I. ELEMENTS OF SYNTHETICAL MECHANICS.

Mechanics is the basis of every course of Natural Philosophy, and the source of all rational explanation of physical phenomena. To the generality of students it has been a forbidden subject, by reason of a want of sufficient preparation in the calculus for its successful study. The aim of the above work is to bring this groundwork of physics within the reach of every one at all acquainted with the merest elements of mathematics.

II. ELEMENTS OF ANALYTICAL MECHANICS.

By the same author. This is the same subject, but treated from a more elevated point of view. It assumes, on the part of the student, an acquaintance with the higher mathematics; and, in reference to its merits, we quote from *Silliman's American Journal of Science:* —

"Professor Bartlett has presented the American student with a very complete and excellent introduction to the Science of Mechanics; a work in which the analytical or deductive method of investigation is systematically carried out, and is, at the same time, set forth in such a manner as to be intelligible to those who possess a fair knowledge of the calculus, has long been needed. We believe Professor Bartlett's work is the only treatise in our language in which this has been attempted."

"THE ELEMENTS OF ANALYTICAL MECHANICS, by Professor W. H. C. Bartlett, is a valuable contribution to our means of obtaining a competent knowledge of this branch of science. In it he has deduced the laws of the movements of bodies on the principles of virtual velocities, instead of the parallelogram of forces, which is made the basis in most English and some French treatises. Combining this with D'Alembert's principle, which is shown to be but a generalization of the Newtonian law of the equality of action and reaction, he deduces six equations for the motions of all bodies, and which contain the whole subject of Mechanics. It is a method susceptible of the most simple, precise, and prolific developments. It places the most vast and fruitful principles of science within the grasp of the tyro, and enables him to commune face to face with the great masters of Mechanics, with Lagrange and Laplace, with Newton and Euler, with Huggens and Bernouilli." — *Putnam's Monthly Magazine, April*, 1856, p. 439.

III. SPHERICAL ASTRONOMY.

By the same author. The aims and merits of this work are set forth in the following notices, also taken from *Silliman's Journal of Science:* —

"This work is worthy of the present state of science, and, as a text-book for the higher classes in colleges, it has no equal in this country, and perhaps none in the language. The peculiarities of the work seem to be these: instead of the geometrical explanations, to which we have been accustomed in our astronomical text-books, of various phenomena, such as the tides, the stations and retrogradations of the planets, their phases, and the changes of the seasons, the author deduces the effects analytically, and the explanation is contained with great neatness in the analytical formulas and their interpretation. The elements of the planetary orbits are deduced with much conciseness and beauty, the more difficult investigations being made in the Appendix, and their results introduced in the text. The great improvements of modern science in this particular are here brought within the reach of every diligent student."

IV. ACOUSTICS AND OPTICS.

By the same author. Of this work, Professor F. A. P. Barnard, the highest authority on all educational matters, says: "It is the most concisely luminous treatise on the subject in the language."

"The student is here presented with the second volume of a course of Natural Philosophy intended for the classes in colleges and universities. The work complete will extend to three volumes, of which the first, embracing Mechanics, has already been noticed in the columns of this journal. The book before us treats of Acoustics, including the velocity, divergence, and reflection of sound; music, chords, intervals, and harmony; and Optics, exhibiting the laws of reflection and refraction; the telescope, microscope, camera-obscura, and polarization of light. Emanating from the proud seat of science, the Polytechnic School of North America, such works need not the blazonry of praise nor the guerdon of critical encomium. The high standard of pure and mixed science at West Point, and the acknowledged excellence of Professors, constitute our best guaranty of merit." — *National Intelligencer.*

A. S. BARNES & CO., 51 and 53 John Street, New York,

Publishers of the National Series of Standard School Books.

COLLEGE TEXT-BOOKS,

PUBLISHED BY A. S. BARNES & CO., NEW YORK.

DAVIES' SYSTEM OF MATHEMATICS.
CHURCH'S CALCULUS.
CHURCH'S ANALYTICAL GEOMETRY.
COURTENAY'S CALCULUS.
HACKLEY'S TRIGONOMETRY.
DAY'S ART OF RHETORIC.
BOYD'S KAMES' ELEMENTS OF CRITICISM.
DWIGHT'S GRECIAN AND ROMAN MYTHOLOGY.
DAVIES' DICTIONARY OF MATHEMATICS.

THE NATIONAL SERIES OF MATHEMATICS.

BY CHARLES DAVIES, LL. D.,

PROFESSOR OF MATHEMATICS IN COLUMBIA COLLEGE, N. Y.

PUBLISHED BY A. S. BARNES & CO., 51 & 53 JOHN STREET, NEW YORK.

Elementary Course.

	Retail price.
DAVIES' PRIMARY ARITHMETIC & TABLE BOOK.	$0 15
DAVIES' INTELLECTUAL ARITHMETIC.	0 25
[DAVIES' FIRST LESSONS IN ARITHMETIC.]	0 20
DAVIES' NEW SCHOOL ARITHMETIC.	0 45
KEY TO DAVIES' NEW SCHOOL ARITHMETIC.	0 45
DAVIES' NEW UNIVERSITY ARITHMETIC.	0 75
KEY TO DAVIES' NEW UNIVERSITY ARITHMETIC.	0 50
[DAVIES' GRAMMAR OF ARITHMETIC.]	0 30
DAVIES' ELEMENTARY ALGEBRA.	0 75
KEY TO DAVIES' ELEMENTARY ALGEBRA.	0 50
DAVIES' ELEMENTARY GEOMETRY & TRIGONOM.	1 00
DAVIES' PRACTICAL MATHEMATICS.	1 00

Advanced Course.

	Retail price.
DAVIES' UNIVERSITY ALGEBRA.	$1 25
DAVIES' BOURDON'S ALGEBRA.	1 50
KEY TO DAVIES' BOURDON'S ALGEBRA.	1 50
DAVIES' LEGENDRE'S GEOMETRY.	1 50
DAVIES' ELEMENTS OF SURVEYING.	1 50
DAVIES' ANALYTICAL GEOMETRY.	1 25
DAVIES' DIFFERENTIAL & INTEGRAL CALCULUS.	1 25
DAVIES' DESCRIPTIVE GEOMETRY.	2 00
DAVIES' SHADES, SHADOWS, AND PERSPECTIVE.	2 50
DAVIES' LOGIC OF MATHEMATICS.	1 25
DAVIES' MATHEMATICAL DICTIONARY.	2 50

A. S. BARNES & Co. have the pleasure of announcing, that they have just issued from their press AN ENTIRELY NEW WORK ON ALGEBRA, by PROFESSOR DAVIES, entitled

DAVIES' UNIVERSITY ALGEBRA.

This work is designed to occupy an intermediate place between his Elementary Algebra and Bourdon. It teaches the Science and Art of Algebra by a logical arrangement and classification of the principles in their natural order, and by illustrating their application in an extended series of carefully arranged and graded examples. It is well adapted for use in High Schools, Academies, and Colleges; the work being so divided and arranged that it may be studied in parts, or as a whole, forming a full and complete course.

Especial attention is invited to DAVIES' NEW SERIES OF ARITHMETICS.

NATURAL PHILOSOPHY AND CHEMISTRY.

PUBLISHED BY A. S. BARNES & CO., NEW YORK.

NATURAL PHILOSOPHY.

PARKER'S JUVENILE PHILOSOPHY.	Price $0 25
PARKER'S FIRST LESSONS IN PHILOSOPHY.	0 37½
PARKER'S COMPENDIUM OF SCHOOL PHILOSOPHY.	1 00

The present edition of Parker's School Philosophy has been corrected, enlarged, and improved, and contains all the late discoveries and improvements in the science up to the present time.

It contains engravings of the Boston School set of apparatus; a description of the instruments, and an account of many experiments which can be performed by means of the apparatus, — and it is peculiarly adapted to the convenience of study and recitation. The work is immensely popular, and in very extensive use, more so than any other work of the kind. *It has been recommended by the Superintendents of Public Instruction of six States, and is the Standard Text-Book in all the principal cities of the United States, and throughout Canada West.*

PORTER'S SCHOOL CHEMISTRY.

First Book of Chemistry, and Allied Sciences, including an Outline of Agricultural Chemistry. By PROF. JOHN A. PORTER. Price 50 cts.

Principles of Chemistry, embracing the most recent discoveries in the Science, and the Outlines of its application to Agriculture and the Arts — illustrated by numerous experiments newly adapted to the simplest apparatus. By JOHN A. PORTER, A. M., M. D., Professor of Agricultural and Organic Chemistry in Yale College. Price $1.

These works have been prepared expressly for Public and Union Schools, Academies, and Seminaries, where an extensive course of study on this subject and expensive apparatus were not desired, or could not be afforded. A fair, practical knowledge of Chemistry is exceedingly desirable, and almost a necessity at the present day, but it has been taught in very few Public or Union Schools, owing entirely to the want of suitable text-books adapted to simple apparatus, or such as could be readily obtained. It is confidently believed that these works supply this great want, and will be found in every respect just what is required. Boxes containing all the apparatus and materials necessary to perform all the experiments described in these books, can be obtained for $8.00, by addressing A. S. BARNES & Co., New York.

SCIENTIFIC BOOKS,

IMPORTED AND FOR SALE BY

JOHN BARTLETT,...CAMBRIDGE.

*** Orders for the Importation of English, French, and German Books will be promptly executed, and at moderate prices.

BOOKS FOR PUBLIC LIBRARIES IMPORTED FREE OF DUTY.

ADHEMAR. Le Traité des Ponts biais. 8vo, et atlas in fol. $8.00.

ANNALES DE L'OBSERVATORIE IMPERIAL DE PARIS. Par Le Verrier. 3 vols. in 4to, hf. cf. $24.00.

ARMSTRONG. Steam Boilers. 30c.

AUDÉ. Poussée des Terres. 8vo, hf. cf. $1.87.

BAKER. Treatise on Statics and Dynamics. 30c.

———. Treatise on Land Surveying and Engineering. 30c.

BARLOW. On Materials and Construction. 8vo. $4.80.

BIOT. Refractions Atmospheriques. 4to. $1.25.

———. Traité des Equations differentielles. Hf. mor. $2.00.

BLACK. Iron Highways from London to Edinburgh, &c. $1.00.

BLAND. Algebraical Problems. 8vo. (Second-hand.) $1.50.

———. Key to do. 8vo. " " $1.75.

———. Philosophical Problems. 8vo. " " $1.50.

BOOTH. Theory of Elliptic Integrals. 8vo. $2.25.

BORDEN. System of Railway Formulæ. 8vo. $2.25.

BOURCHARLAT. Elements de Calcul differential, et du Calcul integral. 8vo. $2.25.

BOURDON. Application de l'Algébre a la Géométrie. 8vo, hf. cf. $3.00.

BOURNE. Treatise on the Steam Engine. 4to. $6.00.

BOW. Treatise on Bracing. 8vo. $1.12.

BRANDE. Dictionary of Science, Literature, and Art. 8vo. $7.50.

BRETON. Traité du Nivellement. 8vo. $2.50.

BUCK. Oblique Arches of Bridges. 4to. $3.75.

BURGOYNE. Blasting and Quarrying Stone. 30c.

BURNELL. Treatise on Limes and Cements. 30c.

BURY. Rudimentary Treatise on Architecture. 45c.

CALLET. Tables de Logarithmes. 8vo, hf. cf. $5.00.

CARMICHAEL. Treatise on Calculus of Operations. 8vo. $2.75.

CHASLES. Traité de Géométrie Supérieure. 8vo, hf. cf. $6.00.

CHAUVENET. Trigonometry. 8vo.

CHOQUET. Traité elémentaire d'Algébre. 8vo, hf. cf. $2.00.

CIVIL ENGINEER AND ARCHITECT'S JOURNAL. Monthly. 4to. London. $7.50 per annum, post-paid.

CLARK. Britannia and Conway Tubular Bridges. 2 vols. 8vo, and atlas folio. $28.50.

CLEGG. Architecture of Machinery. 4to. $2.00.

CRELLE. Journal fur die reine und augewandte Mathematik. $4.00 per annum.

COURNOT. Traité elémentaire de la theorie des fonctions, et du Calcul infinitesimal. 2 vols. 8vo. $4.00.

COURTENAY. Calculus. 8vo.

CRESWELL. Treatise on Spherics. 8vo. (Second-hand.) $1.00.

CRESY. On Bridge Building, and Equilibrium of Vaults and Arches. Folio. $10.00.

———. Cyclopædia of Engineering. 8vo. $15.00.

D'AUBUISSON. Treatise on Hydraulics. Translated by Joseph Bennett. 8vo.

DE LA RIVE. Traité d'Electricité theorique et appliquée. 3 vols. 8vo, hf. mor. $9.00.

———. Treatise on Electricity, in Theory and Practice. 3 vols. 8vo. $22.00.

DELISLE. Géométrie Analytique. 8vo, hf. cf. $3.00.

DE MORGAN. Differential and Integral Calculus. 8vo. $2.00.

DEMPSEY. Treatise on Drainage and Sewerage of Towns and Buildings. 45c.

———. Treatise on Drainage and Sewerage of Districts and Lands. 30c.

DOBSON. On Foundations and Concrete Works. 30c.

DOBSON. On Masonry and Stonecutting. 60c.

———. On Bricks and Tiles. 30c.

DUHAMEL. Cours d'Analyse Calcul infinitésimal. 2 vols. 8vo, pp. 300; hf. mor. $5.00.

———. Cours de Mecanique. 2 vols. 8vo, pp. 300, hf. mor. $4.50.

DUPIN. Application de Géométrie et de Mecanique à la Marine aux Ponts et Chaussées. 4to, hf. mor. $4.50.

———. Developpments de Géométrie. 4to, hf. mor. $4.50.

———. Géométrie et Mecanique, appliquée aux arts. 8vo, hf. cf. $2.50.

EARNSHAW. Dynamics. 8vo. (Second-hand.) $1.50.

EMY. Traité de la Charpenterie. 2 vols. in 4to, et atlas in fol. hf. mor. $25.00.

FAIRBAIRN. On the Application of Cast and Wrought Iron to Building Purposes. 8vo. $3.50.

———. Useful Information for Engineers. 8vo. $3.25.

FERGUSON. Handbook of Architecture. 2 vols. 8vo, hf. cf. $12.

FISCHER. Logarithmic Tables of Seven Places. Translated from Bremiker's Vega. 8vo, hf. mor. $2.50.

FRANCŒUR. Cours complet de Mathematiques pures. 2 vols. 8vo. $5.00.

GANOT. Traité elémentaire de Physique. 12mo. $1.75.

GARBETT. Principles of Design in Architecture. 60c.

GAUSS. Méthode des moindres carrés. 8vo, hf. cf. $1.62.

———. Theoria Motus. Translated by C. H. Davis. 4to. $5.00.

GAYFFIERS. Manual des Ponts et Chaussées. 2 vols. 18mo, hf. cf. $3.00.

GILLESPIE. Land Surveying. 8vo.

———. Roads. 8vo.

GODFRAY. The Lunar Theory. 8vo. $1.62.

GRANT. Plane Astronomy. 8vo. $1.75.

GREGORY. Differential and Integral Calculus, by Walton. 8vo. $5.50.

HADDON. Differential Calculus. 30c.

SCIENTIFIC BOOKS.

HAMILTON. Lectures on Quaternions. 8vo. $6.30.
HANN. Integral Calculus. 30c.
HANN AND GENNER. On the Steam Engine. 8vo. $2.75.
HART. On Oblique Arches. 4to. $2.40.
HASKOLL. Railway Construction, from the setting out of the centre line to the completion of the work. 2 vols. Roy. 8vo. Plates. $12.50.
HEATHER. Descriptive Geometry. 30c.
HEMMING. Differential and Integral Calculus. 8vo. $2.75.
HENCK. Fieldbook for Engineers. 18mo.
HUGHES. Treatise on Gas Works. 90c.
HUTTON. Mathematics. 8vo. $4.00.
———. Recreations. 8vo. $4.00.
HUTTON. Mathematical Tables. 8vo. $3.75.
HYMER. Plane and Spherical Trigonometry. $2.50.
———. Algebraic Equations. 8vo. $3.00.
———. Analytical Geometry. $3.00.
———. Differential Equations, and Calculus of Finite Differences. 8vo. $3.60.
———. Integral Calculus. 8vo. $3.00.
———. Geometry of Three Dimensions. 8vo. Cloth. $3.00.
———. Conic Sections. 8vo. (Second-hand.) $1.25.
JAMIESON. Solutions of the Senate House Rider. 8vo. $2.25.
JELLETT. Calculus of Variations. 8vo. $4.50.
KÖHLER. Logarithmisch Trigonometrisches Handbuch. 8vo. $2.12.
LACROIX. Traité du Calcul Differential et du Calcul Integral. 3 vols. in 4to, avec 18 pl. Mor. fine copy. $35.00.
LAGRANGE. Mecanique Analytique. 2 vols. 4to, hf. mor. $13.00.
———. Theorie des Fonctions Analytiques. 4to, hf. mor. $6.00.
———. Traité des Equations Numeriques de tous les degrés. 4to, hf. mor. $5.00.
LALANDE. Tables de Logarithmes. 18mo. 60c.
LAPLACE. Systeme du Monde. 4to, hf. mor. $5.50.
LAW. Civil Engineering. $1.37.
——. Constructing and Repairing Roads. 30c.
LEGENDRE. Traité des Fonctions Elliptiques. 3 vols. 4to, hf. mor. $20.00.
LEROY. Traité de Stéréotomie comprenant les Applications de la Géométrie Descriptive a la Theorie des Ombres, la Perspective Lineaire, la Gnomonique, la Coupe des Pierres et la Charpente. In 4to, avec atlas de 74 pl. in fol. Paper, $9.00. Hf. mor. $12.00.
———. Traité de Géométrie Descriptive. In 4to, avec atlas de 71 pl. Paper, $3.00. Hf. mor. $5,00, $6.00.
———. Analyse Appliquée a la Géométrie des trois dimensions. 8vo, hf. cf. $2.25.
LIOUVILLE. Journal de Mathematiques pures et appliquées. 1836–1857. 22 vols. 4to, hf. cf. $150.00.
LOUDON. Cyclopædia of Architecture. 8vo. $10.
MAHAN. Civil Engineering. 8vo.
MAIN AND BROWN. Indicator and Dynamometer. $1.40.
MONGE. Application de l'Analyse a la Geometrie. 4to, hf. mor. $10.50.
———. Traité éllementaire de Statique. 8vo, hf. mor. $1.75.
MORIN. Aide-memoire de Mecanique Pratique. 8vo. $2.00.
MOSELY. Mechanics of Engineering. 8vo.
MULLER. Physics and Meteorology. 8vo. $4.00.
NAVIER. (Ketten) Hängbrüchen. 4to. $5.00.
NEVILLE. Hydraulic Formulæ. 8vo. $2.75.
NICOL. Cyclopædia of the Physical Sciences. 8vo. $3.75.
OLIVIER. Theorie géométrique des Engrenages. 4to, hf. mor. $3.75.
———. Developpements de Géométrie Descriptive. 2 vols. 4to, hf. mor. $9.00.
———. Complément de Géométrie Descriptive. 2 vols. 4to, hf. mor. $9.00.
———. Mémoires de Géométrie Descriptive, Theorique, et Appliquée. 2 vols. 4to, hf. mor. $9.00.
———. Application de la Géométrie Descriptive. 2 vols. 4to, hf. mor. $10.00.
PARKINSON. Elementary Mechanics. Post 8vo. $2.87.
PEIRCE. Analytic Mechanics. 4to, cloth. $7.50.
———, J. M. Analytic Geometry. 8vo. $1.50.
PHEAR. Elementary Hydrostatics. Post 8vo. $1.62.
POINSOT. Eléments de Statique. 8vo, hf. cf. $2.50.
POISSON. Mecanique. 2 vols. 8vo. $6.00.
PONCELET ET LESBROS. Experiences Hydrauliques. 4to, hf. mor. $4.00.
PONTECOULANT. Systeme du Monde. 4 vols. 8vo, hf. cf. $16.00.
POUILLET. Elements de Physique Experimentale. 3 vols. 8vo, hf. cf. $7.50.
PRALY. Construction des Voutes biaises. 8vo, hf. cf. $1.87.
PRICE. Differential and Integral Calculus. 3 vols. 8vo. $14.50.
QUARTERLY JOURNAL OF PURE AND APPLIED MATHEMATICS. Sylvester. Feriers & Co. $6.00 per annum. Post paid.
———. 1855–8. 8 Nos. $12.00.
SERRET. Cours d'Algébre supérieure. 8vo, hf. cf. $3.50.
SEWELL. Treatise on Steam and Locomotives. 30c.
SIMMS. On the Use of Instruments. 8vo.
SIMMS. On Levelling. 8vo.
SMITH. Linear Drawing. 8vo.
———. Topographical Drawing. 8vo.
SOLUTIONS OF THE CAMBRIDGE PROBLEMS. 1848 to 1851. Ferrers and Jackson. 8vo. $4.50.
———. 1854. Walton and Mackenzie. 8vo. $3.25.
———. 1857. Campion and Walton. 8vo. $2.50.
——— Senate House Riders. Jamieson. 8vo. $2.25.
SPOTTISWOODE. Elementary Theorems relating to Determinants. 4to. $1.50.
STEVENSON. Treatise on Lighthouses. 90c.
SWINDELL. Well Digging and Boring. 30c.
TATE. Exercises on Mechanics. With Key. 2 vols. $1.62.
———. Mechanical Philosophy. 8vo. $3.25.
———. Materials. 8vo. $1.75.
TATE AND STEEL. Dynamics. Post 8vo. $3.00.
TERQUEM. Nouvelles Annales de Mathématiques. Monthly. $4.00 per annum.
———. Nouvelles Annales de Mathématiques, 1856–7. 2 2 vols. Hf. calf. $6.50.
TODHUNTER. Analytical Statics. Post 8vo. $3.00.
———. Differential and Integral Calculus. 2 vols. Post 8vo. $6.00.
TREDGOLD. Strength of Materials. Edited by Hodgkinson. 2 vols. 8vo. $7.50.
TRESCA. Géométrie Descriptive. 8vo, hf. mor. $3.00.

SCIENTIFIC BOOKS.

URE. Dictionary of Arts, Manufactures, and Mines. 2 vols. 8vo. $5.00.

VALLEE. Traité de la Science du Desin. Contenant la Theorie Générale des Ombres, la Perspective Lineaire, la Theorie Générale des Images d'optique et le Perspective Aérienne appliquée au Lavis, pour faire suite a la Géométrie Descriptive. In 4to, et atlas de 56 pl. Hf. mor. $5.50.

VERHULST. Traite élementaire des Fonctions Elliptiques. 8vo, hf. cf. $2.75.

VIEILLE. Cours complémentaire d'Analyse et de Mécanique. 8vo, hf. cf. $2.50.

VICAT. On Cements. Translated by Capt. Smith.

WALTON. Problems illustrative of Plane Coördinate Geometry. 8vo. $4.80.

———. Problems, Mechanical Collection of. 8vo. $5.50.

———. Problems in Hydrostatics and Hydrodynamics. 8vo. $3.00.

———. On the Differential Calculus. 8vo. $3.25.

WALTON AND MACKENZIE. Solutions of the Cambridge Problems. 8vo. 1854, $3.25. do. 1857, $2.50.

———. Problems in Elementary Mechanics. 8vo, cloth. 1858. $3.00.

WARR. Dynamics, Construction of Machinery. 8vo. $2.87.

WHEWELL. Analytical Statics. 8vo. $2.25.

WIESBACH. Mechanics. 2 vols. 8vo.

WILLIAMS. Practical Geodosy. Post 8vo. $1.50.

WILME. Handbook for Mapping, Engineering, &c. 4to, plain, $7.00. Colored, $12.00.

WILSON'S DYNAMICS. 8vo. $2.87.

WOLFER. Tabulæ Reductionum Observationum Astronomicarum. 8vo. $3.50.

WOOD. Practical Treatise on Railroads. 8vo, hf. mor. $4.00.

WOOLHOUSE. Weights and Measures of all Nations. 30c.

———. Differential Calculus. 30c.

WORTHEN. Appleton's Handbook of Drawing. 8vo.

YVON, VILLARCEAU. Etablissement des Arches de pont. 4to. hf. mor. $5.00.

*** Early copies of all the new mathematical books published in England and France will be received as soon as published.

LIST OF BOOKS PUBLISHED BY JOHN BARTLETT.

TEXT-BOOKS USED IN HARVARD COLLEGE.

THE ORATION OF ÆSCHINES AGAINST CTESIPHON. With Notes, by Professor J. T. Champlin. 12mo, cloth. 88 cts.

THE CLOUDS OF ARISTOPHANES. With Notes, by Professor C. C. Felton. 12mo, cloth. $1.00.

THE BIRDS OF ARISTOPHANES. With Notes, by Professor C. C. Felton. 12mo, cloth. $1.00.

THE PANEGYRICUS OF ISOCRATES. With Notes, by Professor C. C. Felton. 12mo, cloth. 75 cents.

SELECTIONS FROM THE GREEK HISTORIANS. With Notes, by Professor C. C. Felton. 12mo, half mor. $1.50.

SELECTIONS FROM MODERN GREEK WRITERS, IN PROSE AND POETRY. With Notes, by Professor C. C. Felton. 12mo, cloth. $1.00.

THE AIAS OF SOPHOKLES. With Critical and Explanatory Notes, by J. B. M. Gray. 12mo, cloth. $1.25.

PRONUNCIATION AND HISTORY OF THE GREEK ALPHABET. By E. A. Sophocles, Tutor in Harvard College. 12mo, cloth. $1.00.

CICERO, M. T. BRUTUS. Edited, with Notes, by Professor C. Beck. 16mo, cloth. 75 cents.

CICERO. Immortality of the Soul; The Tusculan Disputations, Book First; The Dream of Scipio; and Extracts from the Dialogues on Old Age and Friendship. With English Notes, by Thomas Chase, Tutor in Harvard College. 16mo, cloth. 75 cents.

COOKE'S CHEMICAL PROBLEMS AND REACTIONS, to accompany Stockhardt's Principles of Chemistry. By Professor Josiah P. Cooke. 75 cents.

CONSTITUTIONAL DOCUMENTS OF ENGLAND AND AMERICA, FROM MAGNA CHARTA TO THE FEDERAL CONSTITUTION OF 1789. Compiled and Revised, with Notes, by F. Bowen, Professor of Moral Philosophy. 8vo, cloth. 88 cents.

HODGES, R. M. PRACTICAL DISSECTIONS. By Richard M. Hodges, M. D., Demonstrator of Anatomy in the Medical Department of Harvard College. 16mo. $1.00.

HORACE. With Notes, by A. J. Macleane, M. A. Revised and edited by R. H. Chase, A. M. With Introduction to the Metres, by Professor Charles Beck. Revised edition. 12mo, half mor. $1.25.

LATHAM'S ELEMENTARY ENGLISH GRAMMAR. Revised by Professor Child. With an Appendix, by President Goodwin, Trinity College. 16mo, cloth. 75 cents.

PEIRCE, J. M. ANALYTIC GEOMETRY. 8vo. $1.50.

REID, THOMAS. ESSAYS ON THE INTELLECTUAL POWERS. With Notes and Illustrations, by Sir William Hamilton, and others. Edited by James Walker, D. D., President of Harvard College. 12mo, cloth. $1.25.

ROELKER, B. A GERMAN READER FOR BEGINNERS. Compiled by Bernard Roelker, A. M., Instructor in Harvard University. 12mo, cloth. $1.00.

STEWART, DUGALD. ACTIVE AND MORAL POWERS OF MAN. Edited, with Notes, by James Walker, D. D., President of Harvard College. 12mo, cloth. $1.25.

STOCKHARDT'S PRINCIPLES OF CHEMISTRY. Illustrated by Simple Experiments. Translated by C. H. Peirce, M. D. 12mo, cloth. $1.75.

VERNON, EDWARD JOHNSTON, B. A. GUIDE TO THE ANGLO-SAXON TONGUE. 12mo, cloth. $1.25.

WHATELY. LESSONS ON MORALS AND CHRISTIAN EVIDENCES. Edited by Rev. F. D. Huntington. 16mo. 75 cents.

MISCELLANEOUS.

A COLLECTION OF FAMILIAR QUOTATIONS, WITH INDICES OF AUTHORS AND SUBJECTS. 16mo, cloth. Third edition. $1.00.

AUTUMN LEAVES. ORIGINAL PIECES IN PROSE AND VERSE. 16mo, cloth. 75 cents.

COLLEGE WORDS AND CUSTOMS. By B. H. Hall. 12mo, cloth. $1.25.

FOUR OLD PLAYS. JACK JUGLER, THERSYTES, PARDONER AND FREERE, AND JOCASTA. 12mo, cloth. $1.25.

MEMORIAL OF REV. JOHN S. POPKIN, D. D. Edited by Cornelius C. Felton. 16mo, cloth. $1.25.

MEMORIES OF YOUTH AND MANHOOD. By Sidney Willard. 2 vols. 16mo, cloth. $2.00.

NORTON, ANDREWS. EVIDENCES OF THE GENUINENESS OF THE GOSPELS. Second edition. In three volumes. 8vo, cloth. $5.00.

NORTON, ANDREWS. TRACTS CONCERNING CHRISTIANITY. 8vo, cloth. $1.75.

PROMETHEUS AND AGAMEMNON OF SOPHOCLES. A Translation, by W. H. Herbert. 12mo, cloth. 75 cents.

TUCKERMAN, EDWARD. SYNOPSIS OF NORTHERN LICHENES. 8vo, cloth. 75 cents.

TUCKERMAN, EDWARD. ENUMERATION OF NORTH AMERICAN LICHENES. 12mo, cloth. 87 cents.

SCIENTIFIC AND EDUCATIONAL WORKS,

PUBLISHED BY

GOULD AND LINCOLN,

59 Washington Street, Boston.

HUGH MILLER'S WORKS.

THE OLD RED SANDSTONE; or, New Walks in an Old Field. To which is appended a Series of Geological Papers read before the Royal Physical Society of Edinburgh. A new, improved, and enlarged edition. 12mo, cloth. $1.25.

The new matter in this edition consists of over *one hundred pages* on the following subjects: Geological Evidences in Favor of Revealed Religion; on the Ancient Grauwacke Rocks of Scotland; on the Red Sandstone, Marble, and Quartz Deposits of Assynt; on the Corals of the Oölitic System of Scotland; on the Fossiliferous Deposits of Scotland. The volume embraces also four additional plates, several new cuts, and an Appendix of new Notes. New engravings of the previous illustrations have also been made.

THE FOOT-PRINTS OF THE CREATOR; or, the Asterolepis of Stromness, with numerous Illustrations. With a Memoir of the Author, by LOUIS AGASSIZ. 12mo, cloth. $1.00.

MY SCHOOLS AND SCHOOLMASTERS; or, the Story of My Education. AN AUTOBIOGRAPHY. With a full length Portrait of the Author. 12mo, cloth. $1.25.

FIRST IMPRESSIONS OF ENGLAND AND ITS PEOPLE. With a Likeness of Author. 12mo, cloth. $1.00.

THE TESTIMONY OF THE ROCKS; or, the Bearings of Geology on the Two Theologies, Natural and Revealed. With 152 Illustrations. To which is prefixed Memorials of the Author, embracing an authentic and minute account of his Death, etc. 12mo. pp. 516. $1.25.

THE CRUISE OF THE BETSEY; or, a Summer Ramble among the Fossiliferous Deposits of the Hebrides. With Rambles of a Geologist; or, Ten Thousand Miles over the Fossiliferous Deposits of Scotland. 12mo, cloth. pp. 524. $1.25.

"Miller's descriptions in this work are fresh, eloquent, and true, as any that have ever issued from his pen. We have renewed our recollection of them with infinite pleasure." — *Edinburgh Review.*

THE PLURALITY OF WORLDS. With an Introduction by EDWARD HITCHCOCK, LL.D., and a SUPPLEMENTARY DIALOGUE by the Author, in which his Reviewers are Reviewed. 12mo, cloth. $1.00.

This work is now known to be from the pen of Prof. Whewell, of Cambridge University, England.

"The 'Plurality of Worlds' has created as great a sensation in the reading world as did the 'Vestiges of Creation.'" — *London Cor. of the N. Y. Tribune.*

ANNUAL OF SCIENTIFIC DISCOVERY FOR 1859; or, Year-Book of Facts in Science and Art, exhibiting the most important Discoveries and Improvements in Mechanics, Useful Arts, Natural Philosophy, Chemistry, Astronomy, Meteorology, Zoölogy, Botany, Mineralogy, Geology, Geography, Antiquities, &c., together with a List of recent Scientific Publications; a classified List of Patents; Obituaries of eminent Scientific men; an Index of Important Papers in Scientific Journals, Reports, &c. Edited by DAVID A. WELLS, A. M. With a Portrait of Prof. O. M. Mitchell. 12mo, cloth. $1.25. *Ready in February.*

VOLUMES OF THE SAME WORK for 1850, 1851, 1852, 1853, 1854, 1855, 1856, 1857, 1858. With Portraits of Professors Agassiz, Silliman, Henry, Bache, Maury, Hitchcock, Wyman, Rogers, and Richard M. Hoe. Nine volumes, 12mo, cloth. $1.25 per vol.

This work, issued annually, contains all important facts discovered or announced during the year.

☞ Each volume is distinct in itself, and contains *entirely new matter.*

NATURAL HISTORY OF THE HUMAN SPECIES; its Typical Forms and Primeval Distribution. By CHAS. HAMILTON SMITH. With an Introduction containing an Abstract of the Views of Blumenbach, Prichard, Bachman, Agassiz, and other writers of repute. By SAMUEL KNEELAND, Jr., M. D. With elegant Illustrations. 12mo, cloth. $1.25.

TEXT-BOOKS.

MENTAL PHILOSOPHY, including the Intellect, Sensibilities, and Will. By JOSEPH HAVEN, Prof. of Intel. and Moral Philos. in Amherst College. Royal 12mo, cloth. $1.50.

From Prof. Bowen, Harvard College.

"It is at once a Compend of Philosophy and a History of Philosophy. . . . . Prof. Haven's work deserves high praise; it is well written, well arranged, accurate in information, and sound in opinion."

LECTURES ON METAPHYSICS. By Sir WILLIAM HAMILTON, Bart., late Prof. of Logic and Metaphysics in the University of Edinburgh. Edited by Rev. H. L. MANSEL, B. D., of Oxford University, and JOHN VEITCH, M. A., of Edinburgh. One vol. 8vo. *Ready in January.*

PRINCIPLES OF ZOOLOGY; Touching the Structure, Development, Distribution, and Natural Arrangement of the RACES OF ANIMALS, living and extinct, with numerous illustrations. For the use of Schools and Colleges. Part. I. COMPARATIVE PHYSIOLOGY. By LOUIS AGASSIZ and AUGUSTUS A. GOULD. Revised edition. 12mo, cloth. $1.00.

"It is not a mere book, but a work — a real work in the form of a book. Zoölogy is an interesting science, and here is treated with a masterly hand. It is a work adapted to colleges and schools, and no young man should be without it." — *Scientific American.*

THE EARTH AND MAN. Lectures on COMPARATIVE PHYSICAL GEOGRAPHY, in its relation to the History of Mankind. By ARNOLD GUYOT. Translated from the French by Prof. C. C. FELTON. With Illustrations. 12mo, cloth. $1.25.

"Those who have been accustomed to regard Geography as a merely descriptive branch of learning, drier than the remainder biscuit after a voyage, will be delighted to find this hitherto unattractive pursuit converted into a science, the principles of which are definite and the results conclusive." — *North American Review.*

GUYOT'S MURAL MAPS. A Series of elegant Colored Maps, projected on a large scale for the Recitation Room, consisting of a Map of the World, North and South America, Europe, Asia, Africa, &c., exhibiting the Physical Phenomena of the Globe. By Prof. ARNOLD GUYOT. MAP OF THE WORLD, MOUNTED, $10. MAP OF NORTH AMERICA, MOUNTED, $9. MAP OF GEOGRAPHICAL ELEMENTS, MOUNTED, $9. MAP OF SOUTH AMERICA, MOUNTED, $9.

THESAURUS OF ENGLISH WORDS & PHRASES, so classified and arranged as to facilitate the expression of ideas, and assist in literary composition. New and Improved Edition. By PETER MARK ROGET, late Secretary of the Royal Society, London, &c. Revised and Edited, with a List of Foreign Words defined in English, and other additions, by BARNAS SEARS, D. D., President of Brown University. A NEW AMERICAN, from the late stereotype London Edition, with ADDITIONS and IMPROVEMENTS. 12mo, cloth. $1.50.

A TREATISE ON THE COMPARATIVE ANATOMY OF THE ANIMAL KINGDOM. By Profs. C. TH. VON SIEBOLD and H. STANNIUS. Translated from the German, with Notes, Additions, &c., by WALDO I. BURNETT, M. D., Boston. One elegant octavo volume. Cloth. $3.00.

GEOLOGICAL MAP OF THE UNITED STATES AND BRITISH PROVINCES OF NORTH AMERICA. With an explanatory Text, Geological sections and plates of the Fossils which characterize the Formations. By JULES MARCOU. Two volumes, octavo, cloth. $3.00.

☞ The Map is elegantly colored, and done up with linen cloth back, and folded in octavo form, with thick cloth covers.

THE LIFE OF JOHN MILTON. Narrated in connection with the Political, Ecclesiastical, and Literary History of his time. By David MASSON, M. A., Prof. of English Lit. Univ. Coll., London. Vol. First, *ready in January.*

MATHEMATICAL WORKS,

By GEORGE R. PERKINS, LL. D.,

-LATE PRINCIPAL AND PROFESSOR OF MATHEMATICS IN THE NORMAL SCHOOL OF THE STATE OF NEW YORK.

ARITHMETICAL SERIES.

PERKINS'S ARITHMETICAL SERIES embraces four text-books, which cover the whole ground, from the first lesson of the beginner in counting to the most abstruse and intricate operations embraced in the science. Their distinguishing features, as a whole, and the points on which their claim to superiority rests, are as follows: —

1. They are complete. Nothing connected with the subject is omitted.

2. Each number follows that which precedes it naturally and easily, the step from one to another not being too great for the pupil's comprehension.

3. They are consistent with each other. The definitions and rules in the different numbers are, as far as practicable, in the same words, and similar modes of reasoning are employed throughout.

4. They are philosophically arranged.

5. They are inductive. General laws are deduced from individual cases.

6. They are practical, constructed with direct reference to the wants of the pupil when he shall enter on the actual business of life.

7. Rules and explanations are given tersely. Their point is not lost in a mass of words.

8. They present an unusually large number of examples.

9. The examples, particularly those given first under the rules, do not involve tedious operations.

10. Each rule is illustrated by every variety of example that can fall under it.

11. The examples are so constructed as to require *thought* on the part of the pupil.

12. A principle once taught is not allowed to be forgotten. In one form or other it is made the subject of constant review.

13. Finally, these Arithmetics teach the shortest, simplest, and most easy to be remembered modes of performing the different operations of which they treat.

I. PRIMARY ARITHMETIC. 18mo. 160 pages. Price 21 cents. — This work presupposes no knowledge of Arithmetic. It commences with elementary principles, and lays a sure foundation for what is to follow. From the four fundamental rules it proceeds to Fractions. Next come Decimals and their application to Federal Money.

II. ELEMENTARY ARITHMETIC. 16mo. 347 pages. Price 42 cents. — From the Primary the pupil proceeds to the Elementary, in which it is aimed to discipline the mind, to develop the reasoning powers, and to prepare the pupil for the advanced departments of Mathematics. In the author's treatment of Vulgar Fractions, Percentage, and Interest, his new method of finding the cash balance in Equation of Payments, and his improved mode of Extracting the Cube Root, he has certainly made a great advance on the other Elementary Arithmetics now before the public.

III. PRACTICAL ARITHMETIC. 12mo. 356 pages. Price, Cloth, 62 cents, Paper Sides, 50 cents. — This work covers nearly the same ground as the Elementary, differing from it principally in presenting a greater number of examples. It may, therefore, either follow the Elementary, or be substituted for it. No other work offers the scholar such facilities for practice as this, no less than 3,926 sums being given.

KEY TO PRACTICAL ARITHMETIC. 12mo. 324 pages. Cloth. Price 75 cents.

IV. HIGHER ARITHMETIC. 12mo. 324 pages. Cloth. Price 75 cents. — This is intended as a finishing book for those who would complete a thorough arithmetical course. It embraces all the more abstruse parts of the science, and develops its principles to a greater extent than is usual with school-books on this subject.

PERKINS'S ALGEBRAIC SERIES.

I. ELEMENTS OF ALGEBRA. 12mo. 244 pages. Price 75 cents. — Among the peculiar merits of this work, besides its simplicity, are the conciseness of its rules and definitions; its close and logical reasoning, which calls the powers of the learner into active exercise; and the great number and variety of its examples, which afford every opportunity for extended practice.

II. TREATISE ON ALGEBRA: Embracing, besides the elementary principles, all the higher parts usually taught in Colleges; containing, moreover, the new method of Cubic and higher Equations, as well as the development and application of the more recently discovered Theorem of Sturm. 8vo. Sheep. 420 pages. Price $1.50. — What the Elements are to Common Schools, this Treatise is to Academies and Colleges. It will be seen, from the title given above, that it is comprehensive and complete.

PERKINS'S GEOMETRICAL SERIES.

I. ELEMENTS OF GEOMETRY, with Practical Applications. 12mo. 320 pages. Price $1.00. — In these Elements it is aimed to strip Geometry of its difficulties, and render it an attractive study. This is effected by giving a practical bearing to every thing that is taught. The pupil is not allowed to grope in the dark, and ask, "What is the use of these demonstrations?" As soon as a principle is explained, it is applied to the practical purposes of life by means of remarks, suggestions, and questions, added in smaller type. This original feature invests Geometry with an interest of which its apparently abstract character has heretofore deprived it.

II. PLANE AND SOLID GEOMETRY: to which are added, Plane and Spherical Trigonometry and Mensuration, accompanied with all the necessary Logarithmic and Trigonometric Tables. Large 8vo. 443 pages. Price $1.50. — This work is intended to follow the Elements, and gives an extended course in the higher as well as the more rudimental departments of the science, adapted for advanced schools and colleges. It is based on the admirable work of Vincent, revised by Bourdon, which has long been the geometrical standard in the French schools. All that is valuable in Vincent has been taken; but the mathematical attainments and practical skill of Prof. Perkins are everywhere exhibited in adapting, modifying, rearranging, and adding.

PERKINS'S PLANE TRIGONOMETRY, and its application to Mensuration and Land Surveying, accompanied with all the necessary Logarithmic and Trigonometric Tables. 8vo. 328 pages. Sheep. Price $1.50. — This work is remarkable for its simplicity, and bears throughout the marks of its practical origin. The beginner in the science and the proficient will alike find matter of prime value in its pages. The chapters on Land Surveying will prove of incalculable assistance to those who intend following this pursuit in life. The necessary Tables are furnished in an Appendix.

D. APPLETON & CO., 346 & 348 Broadway, New York.

WORCESTER'S QUARTO DICTIONARY.

PROPOSALS OF

HICKLING, SWAN AND BREWER, BOSTON,

FOR PUBLISHING BY SUBSCRIPTION,

A DICTIONARY OF THE ENGLISH LANGUAGE,

BY JOSEPH E. WORCESTER, LL. D.

ONE VOLUME, QUARTO, LIBRARY EDITION.

We propose to publish a library edition of Worcester's Quarto Dictionary for subscribers. It will be printed on extra fine paper, with large margin, a specimen copy of which may be seen at our counting-room.

The work is now rapidly approaching completion, and we hope to publish it in May, 1859. It will be comprised in about eighteen hundred pages, and will contain a full vocabulary of the words now used in Literature, Art, and Science, together with such local and obsolete terms as are likely to be met with in writings that are now much read.

In Orthography the work will represent the best usage both in this country and in England.

The Pronunciation of all the words will be exhibited by a system of notation which will be easily understood; and with regard to words of various, doubtful, or disputed pronunciation, the best authorities for the different modes will be given.

In the department of Etymology this Dictionary will be found to be more complete and satisfactory than any other work of the kind, giving, in a brief form, the results of the investigations of the best writers on this subject.

The Definitions will be fully and accurately discriminated, distinguished by numbers, and exemplified, whenever practicable, by citations from the best authors. In the selection of examples the aim has been to take such as should be valuable for the thought or sentiment they express, so that this Dictionary will present, in a convenient form for reference, a rich collection of the maxims and gems of the language.

The treatment of Synonymes will form a very valuable feature of the work. Very few, even of the best speakers and writers, become so thoroughly masters of their native language as never to experience embarrassment in discriminating between several expressions nearly related. It is to help in overcoming this difficulty that Dr. Worcester has prepared, in connection with those words which seem most to require it, a notice of the synonymous terms, showing, at a glance, the distinctions to be observed in choosing among them.

The grammatical forms and inflections of words will be given more fully than ever before in any English Dictionary, and brief critical notes on the orthography, the pronunciation, the grammatical form and construction, and on the peculiar technical, local, provincial, and American uses of words, will be found scattered throughout the volume.

The Illustrations by wood-cuts, of which there will be about twelve hundred, beautifully executed, will form another novel and useful feature of this Dictionary. There are many terms, the verbal explanation of which, however carefully made, will convey a much less correct idea of their meaning than a pictorial representation, and accordingly it is proposed to adopt this method of exemplifying the definitions in all such cases as seem to require it.

Much important and useful matter will be given in the Introduction on the following subjects: — The *Principles of Pronunciation; Orthography; English Grammar;* the *Origin, Formation, and Etymology of the English Language; Archaisms, Provincialisms, and Americanisms;* and the *History of English Lexicography;* with a notice of *English Orthoepists,* and a *Catalogue of English Dictionaries of the various Arts and Sciences, Encyclopædias,* &c.

In an Appendix will be added Walker's *Key to the Pronunciation of Classical and Scripture Proper Names,* much enlarged and improved; a *Pronouncing Vocabulary of Modern Geographical Names;* a *Collection of Phrases and Quotations from Foreign Languages; Abbreviations used in Writing and Printing,* &c.

The price of the library edition, on extra fine paper, will be $7.50, which will be the retail price for the common edition. Persons subscribing will therefore secure the library edition at the same price that they would be obliged to pay for the common edition after its publication.

Those persons who desire to become subscribers to the work can do so by application to the publishers.

WORCESTER'S SPELLING-BOOK.

WE ALSO RESPECTFULLY INFORM TEACHERS AND SCHOOL COMMITTEES THAT WE PUBLISH A

PRONOUNCING SPELLING-BOOK OF THE ENGLISH LANGUAGE,

BY J. E. WORCESTER, LL. D.

The orthography and pronunciation of Dr. Worcester's Dictionaries represent the best usage of the English language. The Spelling-Book presents the same system, and Teachers throughout the country will welcome its appearance. It is the most accurate, comprehensive, and complete Spelling-Book ever published. Since its publication it has been introduced into use in the Public Schools of New York, Boston, Charlestown, Washington, D. C., West Roxbury, Thomaston, Waldoboro, Wiscasset, Dedham, Fall River, Newport, &c.

☞ Copies will be furnished by mail for examination, on the receipt of 12 cents in postage stamps.

THE

MATHEMATICAL MONTHLY.

Vol. I....JANUARY, 1859....No. IV.

PRIZE PROBLEMS FOR STUDENTS.

I.

If the triangle DEF be inscribed in the triangle ABC, the circumferences of the circles circumscribed about the three triangles AEF, BFD, CDE, will pass through the same point.

II.

Given the base of a spherical triangle, and the ratio of the tangents of the angles at the base; to find the locus of the vertex.

III.

If in any triangle a line be drawn from the vertex of either angle to the opposite side, bisecting the angle, prove that the product of this line and the secant of half the bisected angle equals a harmonic mean between the two sides containing the bisected angle.

IV.

If A, B, C, and D be any four points in the same plane, so situated that four circles ABC, ABD, ACD, and BCD can be drawn through them three and three, prove that the circumferences of any two of these circles will intersect at the same angle as the circumferences of the remaining two.

V.

A paraboloid of given dimensions, but unknown specific gravity, is immersed in common water, until its summit coincides with the surface of the fluid. The pressure from above being removed, the body ascends by the force of the water until its base coincides with the fluid's surface, and then descends, and so on. Find from this circumstance the specific gravity of the body.

The solutions of these problems must be received by the first of March, 1859.

THE ORDER OF MATHEMATICAL STUDIES.

By Rev. Thomas Hill. Waltham, Mass.

The order in which Nature presents ideas to the infant mind, is the proper order in which those ideas should be systematically developed. And the first mathematical idea that enters a child's mind is that of form; the child recognizes a vast variety of objects by their form before it can count. Geometry is therefore the first mathematical study for a child, and should precede arithmetic.

But theorems and demonstrations are wholly unsuited for a child; geometry must be recognized as food for other powers than those of reasoning. Nature presents forms to the eye, and stimulates the child's conception of figures, years before it is capable even of the simplest process of geometrical reasoning. Geometry should, therefore, in a natural system of education, begin with addressing the eye, and stimulating the powers of observation. Little bricks, Chinese tangrams, rude compasses, blackboard drawings, and similar means of illustrating form and the laws of form to the eye, should be in constant use from an early age. The habit of exactness in laying the bricks and blocks, and of critically comparing

and analyzing figures drawn on the blackboard, early formed, will be a great aid in gaining that quickness and accuracy of observation which is one of the rarest and most valuable of intellectual powers. Number must also be first presented, as in nature, in the concrete form; and the proper time for doing this is evidently to be found at the period when the symmetrical building with bricks, or the symmetrical chalk drawings, have introduced the idea of number as a distinct intellectual element. But the first lessons in number should evidently be concrete, such as may be given by a pint of corn. From the actual separation of numbers into their equal parts by separating the corn into equal heaps, will come the first clear ideas of prime and composite numbers. In like manner the idea of multiplication, and the commutation of factors; of division, and the process of dividing by the quotient to find the divisor; of the rate of increase in powers; and of the rapidity with which numbers increase in decimal progression, can be clearly conveyed in no other way than by beans, counters, or corn.

The next step is to appeal to the imagination, and develop systematically the powers of conception. This is the peculiar office and excellence of geometry, and yet it is a point to which writers on that science have seldom referred. All mathematics, and indeed all studies and occupations of life, require the ability to conceive clearly as a real thing, that which has been described in words. The first study to require and develop this ability is found in simple geometry. But inasmuch as the powers of conception are developed much earlier than those of reasoning, it seems to me proper that a child should be taught to conceive of geometrical truths before it is taught to demonstrate them. They may be presented to him in a logically connected series, and in simple forms of language, not avoiding the scientific names of figures, but carefully avoiding scientific terms in the definition and description of the figure. A judi-

cious selection of geometrical facts and names may be thus stored in the child's memory while you are at the same time giving him a power of quick and accurate conception, which will enable him to solve all ordinary questions of loci at a single glance, without reasoning, but by direct sight. Nor in giving him facts should we confine ourselves to those which may be most readily demonstrated, but rather to those which will most stimulate the imagination, and which will lure him upward with a desire to demonstrate them.

The corresponding period in arithmetic introduces the child to the rules of written arithmetic. The decimal notation in Arabic figures is, of course, the first thing to be learned; and it should be taught, at first, as extending on either side of the unit's place. No advantages, on the contrary great disadvantages, arise from postponing a knowledge of this law on the right of the decimal point, to a later period. The moment that a child is able to understand the meaning of 345 he can also understand 3.45 or .345; and the postponement of an explanation of the latter expressions to a later period, invests them with factitious difficulties, that will impair the pupil's freedom in the use of decimals for many years, if not for life. Prof. Loomis, in his note (Math. Month. p. 73), seems not to have noticed the main point of my remark, and omits my careful qualification "at first" from the phrase on which he comments. At the age of fourteen or fifteen years a child has his reasoning powers somewhat developed, and will begin to relish the demonstration of both arithmetical and of geometrical problems. And herein also the course of nature should be followed. The first essays toward demonstration are usually by nature analytical, in the metaphysical sense of that word, and yet almost all writers on geometry make use almost wholly of synthesis. About the same period of the pupil's life he may begin algebra, at first as an extension of arithmetic, afterward as the law of all magnitude, and especially of unknown or variable elements.

After this period, the order of study becomes not unimportant, but less important, than for a younger child. The powers of observation, if not cultivated in early childhood, are apt to become permanently dulled; and the same is true of the powers of conception. Not only do I find in the primary schools in which geometry is studied, that the scholars of eight to ten years old are quicker in understanding it than those from twelve to fourteen; but I have noticed that the same individual, in passing from the younger to the older period without any cultivation of his geometrical tastes, has lost, in his power of understanding, my isolated experimental lessons. I have, therefore, thought it worth while to occupy thus much room in the pages of the Monthly, to call the attention of teachers to the importance of mathematical training in the earliest years; and more especially to the importance of restoring geometry to its ancient place as the foundation of learning.

PROPOSITIONS RELATING TO THE CONE AND SPHERE.

BY E. S. SNELL.
Professor of Mathematics in Amherst College, Amherst, Mass.

THERE is a right cone, which appears to be as remarkable among cones, as the cylinder of Archimedes among cylinders. It is that whose slant height has to the radius of its base, and consequently its convex surface to the area of its base, the ratio of *three* to *one*. Some of its curious properties I presented about ten years ago in Silliman's Journal. Those, with two or three others since observed, I briefly state without proof.

1. The right cone, whose slant height is to the radius of base as 3 : 1, has the *greatest volume* within a given surface.

2. It has *less entire surface* than any other cone circumscribing the same sphere.

3. It has *less volume* than any other cone circumscribing the same sphere.

4. It has *twice* the *height* of the inscribed sphere.

5. Its *entire surface* is *twice* that of the inscribed sphere.

6. It has *twice* the *volume* of the inscribed sphere.

7. Its circle of contact on the inscribed sphere is so situated, that the surface of the segment *below* it, is *twice* that of the segment *above* it.

8. The half surface of the inscribed sphere is *twice* the area of the base of the cone.

MAGIC SQUARE FOR THE YEAR 1859.

By J. Wiessner. Washington, D. C.

From 109 to 229.

1859.

188	201	214	227	119	121	134	147	160	173	175
176	189	202	215	228	109	122	135	148	161	174
212	225	117	130	132	145	158	171	184	186	199
200	213	226	118	120	133	146	159	172	185	187
127	140	142	155	168	181	194	207	209	222	114
115	128	141	143	156	169	182	195	197	210	223
224	116	129	131	144	157	170	183	196	198	211
151	153	166	179	192	205	218	220	112	125	138
139	152	154	167	180	193	206	208	221	113	126
164	177	190	203	216	229	110	123	136	149	162
163	165	178	191	204	217	219	111	124	137	150

1859. 1859. 1859.

EXTRACTION OF THE HIGHER ROOTS OF NUMBERS.

BY DASCOM GREENE.
Professor of Mathematics in Rensselaer Polytechnic Institute, Troy, N. Y.

In a note on the extraction of the cube root of numbers in No. II. of the Mathematical Monthly, by the editor, an easy method is given of forming each trial divisor from the preceding complete divisor by the addition of numbers already used. Another simple method is as follows: From equations (4) and (5) of that article,

$$\text{1st complete divisor} = 3r_1^2 + 3r_1r_2 + r_2^2,$$

$$\text{2d trial divisor} \quad = 3r_1^2 + 6r_1r_2 + 3r_2^2,$$

$$= 1(3r_1^2) + 2(3r_1r_2) + 3(r_2^2);$$

hence, from any complete divisor we find the next trial divisor by multiplying its first term by 1, the second by 2, the third by 3, and taking the sum.

An advantage of this method is, that it may be extended to the case of any higher root. Thus, N being the given number, and supposing r_1 to have been already found, we shall have

$$N^{\frac{1}{n}} = r_1 + r_2;$$

$$\text{or} \quad N = (r_1 + r_2)^n = r_1^n + nr_1^{n-1}r_2 + \frac{n(n-1)}{1.2}r_1^{n-2}r_2^2 + \&c.$$

$$(1) \quad \therefore r^2 = \frac{N - r_1^n}{nr_1^{n-1} + \frac{n(n-1)}{1.2}r_1^{n-2}r_2 + \frac{n(n-1)(n-2)}{1.2.3}r_1^{n-3}r_2^2 + \&c.}$$

We now have

$$N^{\frac{1}{n}} = r_1 + r_2 + r_3;$$

$$\text{or} \quad N = (r_1 + r_2 + r_3)^n$$

$$= (r_1 + r_2)^n + n(r_1 + r_2)^{n-1}r_3 + \frac{n(n-1)}{1.2}(r_1 + r_2)^{n-2}r_3^2 + \&c.$$

$$(2) \quad \therefore r_3 = \frac{N - (r_1 + r_2)^n}{n(r_1 + r_2)^{n-1} + \frac{n(n-1)}{1.2}(r_1 + r_2)^{n-2}r_3 + \&c.}$$

The trial divisor for finding r_3 is

(3) $$n(r_1+r_2)^{n-1} = n r_1^{n-1} + n(n-1) r_1^{n-2} r_2 + \frac{n(n-1)(n-2)}{1.2} r_1^{n-3} r_2^2 + \&c.,$$

and its terms are evidently formed from those of the complete divisor in (1) by multiplying the first by 1, the second by 2, the third by 3, and so on.

As a first example, let us take that contained in the article above referred to; the operation is as follows.

162467446993496	| 54566 = cube root,		
125	$3 \times 5^2 = 75..$	75..	
37467	$3 \times 5 \times 4 = 60.$	120.	
	$4^2 = 16$	48	
32464	$= 4 \times 8116$	8748..	8748..
5003446	$3 \times 54 \times 5 = 810.$	1620.	
	$5^2 = 25$	75	
4414625	$= 5 \times 882925$	891075..	891075..
588821993	$3 \times 545 \times 6 = 9810.$	19620.	
	$6^2 = 36$	108	
535233816	$= 6 \times 89205636$	89303808..	
53588177496	$3 \times 5456 \times 6 = 98208.$		
	$6^2 = 36$		
53588177496	$6 \times$	8931362916	

As a second example, let it be required to extract the 5th root of 2956466552832.

2956466552832	| 312 = 5th root.		
243	$5 \times 3^4 = 405....$		405....
5264665	$10 \times 3^3 \times 1 = 270...$		540...
	$10 \times 3^2 \times 1^2 = 90..$		270..
	$5 \times 3 \times 1^3 = 15.$		60.
	$1^4 = 1$		5
4329151	$= 1 \times 4329151$	4617605....	
93551452832	$10 \times 31^3 \times 2 = 595820...$		
	$10 \times 31^2 \times 2^2 = 38440..$		
	$5 \times 31 \times 2^3 = 1240.$		
	$2^4 = 16$		
93551452832	$= 2 \times$	46775726416	

We propose hereafter to give an application of this method of extracting roots to the resolution of the higher numerical equations.

INVESTIGATION OF THE NATURE OF THE CURVE OF A DRAWBRIDGE.

BY WILLIAM WATSON.
Tutor in the Lawrence Scientific School, Cambridge, Mass.

THE following investigation was made to determine the equation of the curve which sustains the counterpoise of a drawbridge in such a manner, that the movable platform is in equilibrium in every possible position.

The annexed figure, which is drawn from a model recently presented to the Lawrence Scientific School, represents the bridge in elevation. AB is a vertical projection of the platform, which rotates upon a horizontal axis, projected at A. AC is a post, at the top of which there is a fixed pulley, C. ICE is a chain, attached to the platform at I, passing over the fixed pulley C, and connected with a roller E, which rests upon a rigid curve, CF.

The problem then is, to determine the nature of the curve, such that the platform shall constantly be in a state of equilibrium.

Let C be the origin of polar coördinates $(r\varphi)$; CA the prime radius. Let W be the weight of the platform; W_1 that of the roller. Let c be the length of the chain; l that of the platform. Let the angle BAC be denoted by θ; the angle of r and the corresponding tangent by ε. Let a be the height of the post, the same as AI. Assume A as a centre of statical moments; and consider the curve smooth. The statical moment of the platform about A is the product of its weight by its lever arm, that is, $\frac{1}{2} Wl\sin\theta$; that of the

roller is the product of T, the tension of the chain, by the arm AL; but since the tension T acts to produce rotation in an opposite direction to that produced by W, the moment must be regarded as negative; that is, $-T \times AL$. But

$$T = W_1 \cos(\varphi + \varepsilon) \sec \varepsilon; \text{ and } AL = a \cos \tfrac{1}{2}\theta.$$

$$\therefore -T.AL = -W_1 \cos(\varphi + \varepsilon) \sec \varepsilon . a \cos \tfrac{1}{2}\theta$$

$$= \text{the statical moment of the roller.}$$

Since the bridge is in equilibrium in every position, the sum of the statical moments must be zero.

$$\therefore \tfrac{1}{2} W l \sin \theta - W_1 \cos(\varphi + \varepsilon) \sec \varepsilon \, a \cos \tfrac{1}{2}\theta = 0;$$

or, reducing, $W l \sin \tfrac{1}{2}\theta - W_1 a (\cos\varphi - \sin\varphi \tan\varepsilon) = 0$.

Substituting for $\sin \tfrac{1}{2}\theta$ its value $\frac{c-r}{2a}$, we have

$$W l \left(\frac{c-r}{2a}\right) - W_1 a (\cos\varphi - \sin\varphi \tan\varepsilon) = 0.$$

Substituting for $\tan\varepsilon$ its value $r D_r \varphi$, and B for $\frac{2a^2 W_1}{Wl}$, we have

$$c - r - B(\cos\varphi - \sin\varphi \, r D_r \varphi) = 0,$$

or
$$(c - r - B\cos\varphi) D_\varphi r + B r \sin\varphi = 0.$$

The integral of this equation, since it satisfies the usual conditions of integrability,

$$D_\varphi (c - r - B\cos\varphi) = D_r (B r \sin\varphi),$$

is
$$\int_r (c - r - B\cos\varphi) + \Phi = \text{constant},$$

or
$$cr - \frac{r^2}{2} - B r \cos\varphi + \Phi = \text{constant};$$

Φ being an arbitrary function of φ. To determine Φ we have

$$\Phi = \int_\varphi \left[D_\varphi \left(cr - \frac{r^2}{2} - B r \cos\varphi \right) - B r \sin\varphi \right] = \int_\varphi 0 = \text{constant}.$$

The most general equation of the curve is therefore

$$2cr - r^2 - 2 B r \cos\varphi = \text{constant}.$$

Taking the origin on the curve, the equation reduces to

$$r = 2c \left(1 - \frac{B}{c} \cos\varphi\right).$$

If $B = c$, this equation becomes that of the cardioid referred to its cusp, in which the length of the chain c becomes the radius of the generating, as well as that of the directing circle.

I propose, in some future number of the Monthly, to discuss the general equation.

ON A SIMPLIFICATION IN COMPUTING EARTHWORK.

By J. B. Henck. Civil Engineer, Boston.

When ground of considerable extent has been divided, preparatory to excavation, into equal rectangles, it often happens, that the surface within some of these rectangles rises or falls so much towards the centre, as to make it necessary to take with the level the height at the centre, in addition to the usual heights at the corners. Figure 1 represents such a case. $F\,B\,C\,D\,E$ is the ground surface. B, C, D, E are the corners where heights are taken, F is the point where the additional height is taken, and $F_1\,B_1\,C_1\,D_1\,E_1$ is the bottom of the excavation. The mass of earth between these top and bottom surfaces is bounded by vertical planes, and has, by supposition, for its horizontal section a rectangle. The usual mode of computing the solidity in such a case, is to divide the whole mass into four triangular prisms, each of which has for its solidity the product of its horizontal section by one third the sum of its vertical edges.* Thus if the whole horizontal section be represented by A, and the several vertical edges by the letters attached to

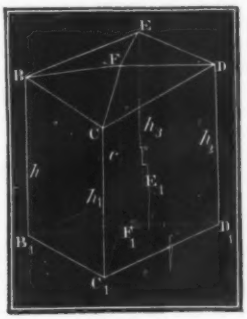


Fig. 1.

* See the author's "Field-Book for Railroad Engineers," p. 93.

them in the figure, we shall have for the solidity of the four triangular prisms,

$$(1) \quad \frac{A}{4} \times \frac{(h+h_1+c)+(h_1+h_2+c)+(h_2+h_3+c)+(h_3+h+c)}{3}.$$

This method is correct, but it compels us to interrupt the usual mode of calculating any number of adjacent prisms by one operation. The exceptional solid has to be calculated alone, and the labor of calculating the surrounding prisms is increased. It is proposed to show, that the exceptional solid may be calculated with the others, just as if no additional height had been necessary, and that a proper correction may be introduced afterwards.

In calculating a number of rectangular prisms by one operation, it is assumed that the solidity of each prism is equal to the product of its horizontal section by one fourth the sum of its vertical edges. Now we can so modify the expression (1) as to separate from it the product just mentioned, namely,

$$\frac{A}{4}(h+h_1+h_2+h_3).$$

For by collecting the like quantities in (1) it becomes

$$\frac{A}{4}\left[\tfrac{2}{3}(h+h_1+h_2+h_3)\right]+\frac{Ac}{3}$$

$$(2) \quad =\frac{A}{4}(h+h_1+h_2+h_3)+\frac{Ac}{3}-\frac{A}{4}\times\tfrac{1}{3}(h+h_1+h_2+h_3)$$

$$=\frac{A}{4}(h+h_1+h_2+h_3)+\frac{A}{3}\left(c-\frac{h+h_1+h_2+h_3}{4}\right).$$

Now, by including this solid in the general calculation with the adjacent prisms, we should take in the first term of (2), namely,

$$\frac{A}{4}(h+h_1+h_2+h_3),$$

leaving for the correction to be made the second term of (2), or

$$\frac{A}{3}\left(c-\frac{h+h_1+h_2+h_3}{4}\right).$$

This correction, it will be seen, represents a pyramid of base A, and altitude equal to the centre height minus the average of the four corner heights, and it is, therefore, very readily obtained. If the centre height were less than the average of the corner heights, the correction would be a minus quantity, as it should be.

Fig. 2.

An exceptional solid sometimes arises in another way. The surface of one of the rectangular divisions may differ considerably from a plane, but may admit of being divided into two planes meeting in one of the diagonals (fig. 2). This would give two triangular prisms, the solidities of which, with the former notation, would be

$$(3) \qquad \frac{A}{2} \times \frac{h + 2h_1 + h_2 + 2h_3}{3}.$$

To combine this solid with the adjacent prisms, we must again be able to separate from (3) the expression $\frac{A}{4}(h + h_1 + h_2 + h_3)$. This can be done, and (3) becomes

$$\frac{A}{4}(h + h_1 + h_2 + h_3) + \frac{A}{12}[(h_1 + h_3) - (h + h_2)].$$

So that we may include this solid also in the general calculation, if we make the correction

$$\frac{A}{12}[(h_1 + h_3) - (h + h_2)].$$

In this correction $h_1 + h_3$ represents the sum of the heights touched by the dividing diagonal, and $h + h_2$ represents the sum of the heights not touched by the diagonal, and the latter sum is always to be subtracted from the former. If, then, the sum of the heights touched by the diagonal is greater than the sum of the other two, the correction will be positive; but if the first sum is less than the second, the correction will be negative.

DEMONSTRATION OF A THEOREM.

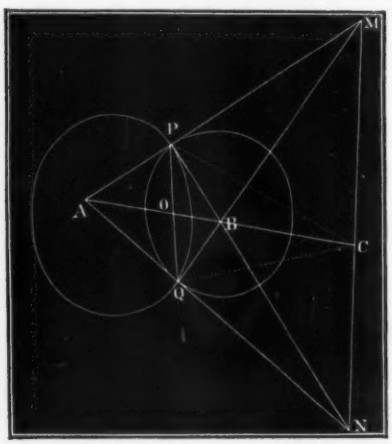
BY A. CAYLEY, F.R.S.
Associate Editor of the London Quarterly Journal of Pure and Applied Mathematics.

SIR, — The annexed demonstration of the very elegant geometrical theorem, Prize Problem No. V.* of the first number of the Mathematical Monthly, may perhaps be interesting to some of your readers, and if you should think it suitable, I shall be glad if you will insert it in your journal. I remain, sir, your obedient servant,

A. CAYLEY.

LONDON, 2 Stone Buildings, W. C., 23d Nov., 1858.

DEMONSTRATION. If, instead of the four circles, we consider any two of the circles, the triangle will be that formed by any three of the four common tangents of the two circles, and the theorem may be thus stated: "Given two circles, consider the triangle formed by any three of the four common tangents of the circles; the circle circumscribed about this triangle will pass through the middle point of the line of junction of the centres of the two circles." A circle may be considered as a conic intersecting the line infinity in two fixed points, the "circular points at infinity:" the centre of the circle is the pole of the line infinity with respect to the circle; the middle point of the line of junction of the two centres is the fourth harmonic with respect to the two centres and the point in which the line of junction meets the line infinity. Consider, then, any two conics intersecting in the points P, Q, and let the tangents at



* By H. A. NEWTON, Professor of Mathematics in Yale College, New Haven, Ct.

P, Q to the first conic meet in the point A, and the tangents at P, Q to the second conic meet in the point B, and let $P\,Q$ meet $A\,B$ in O, and C be the fourth harmonic of O with respect to the points A and B: the point C may also be determined as follows, namely, if the tangent at P to the first conic meet the tangent at Q to the second conic in M; and the tangent at P to the second conic meet the tangent at Q to the first conic in N; then C is the point of intersection of $A\,B$ and $M\,N$. And we have the following theorem, namely: "Given two conics intersecting in the points P and Q; let the point C be determined as above; then considering the triangle formed by any three of the four common tangents of the two conics, the angles of this triangle, and the points P, Q, C lie in one and the same conic;" a theorem which includes the theorem for two circles. And the analytical demonstration is very easy; for take $x = 0, y = 0, z = 0$ as the equations of the lines $A\,B$, $P\,Q$, and $M\,N$ respectively; the equations of the two conics may be written

$$x^2 - (y - z)^2 = 4\,\lambda^2 y^2,$$
$$x^2 - (y + z)^2 = 4\,\mu^2 y^2;$$

where λ, μ are arbitrary coefficients. The equations of the sides $P\,Q$, $Q\,C$, $C\,P$ of the triangle $P\,Q\,C$ would be $y = 0, z - x = 0$, $z + x = 0$ respectively. The equations of the common tangents of the two conics may be found in the ordinary manner, and if we put for shortness

$$\Delta = \sqrt{(\lambda - \mu)^2 + 1},\ \Delta_1 = \sqrt{(\lambda + \mu)^2 + 1},$$

then the equations of the four common tangents are found to be

$$\Delta\, x + (\lambda + \mu) y + (\lambda - \mu)\, z = 0,$$
$$-\Delta\, x + (\lambda + \mu) y + (\lambda - \mu)\, z = 0,$$
$$\Delta_1 x + (\lambda - \mu) y + (\lambda + \mu)\, z = 0,$$
$$-\Delta_1 x - (\lambda - \mu) y + (\lambda + \mu)\, z = 0.$$

The general equation of a conic passing through the angles of the triangle formed by three of these lines, say the first three, is

$$\frac{A}{Ax+(\lambda+\mu)y+(\lambda-\mu)z}+\frac{B}{-Ax+(\lambda+\mu)y+(\lambda-\mu)z}$$
$$+\frac{C}{A_1x+(\lambda-\mu)y+(\lambda+\mu)z}=0;$$

and the theorem will be true, if A, B, C can be so determined that this conic may pass through the angles of the triangle

$$y=0,\quad z-x=0,\quad z+x=0.$$

If we substitute in the equation successively the values

$$y=0,\ z=x;\quad y=0,\ z=-x;\quad x=0,\ z=0;$$

we find

$$\frac{A}{A+\lambda-\mu}+\frac{B}{-A+\lambda-\mu}+\frac{C}{A_1+\lambda+\mu}=0,$$
$$\frac{A}{A-\lambda+\mu}+\frac{B}{-A-\lambda+\mu}+\frac{C}{A_1-\lambda-\mu}=0,$$
$$\frac{A}{\lambda+\mu}+\frac{B}{\lambda-\mu}+\frac{C}{\lambda+\mu}=0;$$

or attending to the values of A and A_1, the equations are

$$A(A-\lambda+\mu)+B(-A-\lambda+\mu)+C(A_1-\lambda-\mu)=0,$$
$$A(A+\lambda-\mu)+B(-A+\lambda-\mu)+C(A_1+\lambda+\mu)=0,$$
$$A(\quad\lambda-\mu)+B(\quad\lambda-\mu)+C(\quad\lambda+\mu)=0;$$

which are obviously equivalent to the two equations

$$AA+B(-A)+CA_1=0,$$
$$A(\lambda-\mu)+B(\lambda-\mu)+C(\lambda+\mu)=0,$$

and give

$$A:B:C=-A(\lambda+\mu)-A_1(\lambda-\mu):-A(\lambda+\mu)+A_1(\lambda-\mu):2A(\lambda-\mu);$$

and the theorem is consequently proved.

NOTES ON THE HYPOCYCLOID.

BY PROF. O. ROOT.

In the first Number of the Mathematical Monthly, Mr. Watson shows that the curve of equilibrium for a heavy rod, with one end against a smooth vertical wall, is the Hypocycloid; in which the radius of the generating circle is one fourth of that of the fixed circle; of which the equation is

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = r^{\frac{2}{3}}, \tag{1}$$

r being one half of the length of the rod. This elegant equation occurs in several problems of interest.

In Leyburn's Mathematical Repository, problem 318 is this: If from one of the angles of a rectangle, a perpendicular be drawn to its diagonal, and from their intersection, lines be drawn perpendicular to the sides containing the opposite angle, we shall have, by denoting these last perpendiculars by x and y, and the diagonal by r, the equation (1).

In the Cambridge problems for 1803, it is required to find the longest ladder which can be slid up a vertical wall under an obstacle given in position. If we denote the coördinates of the obstacle by $(x\,y)$, and the length of the ladder by r, we shall get by the ordinary process, equation (1), which of course gives r when x and y are known. In "Wright's Solutions of the Cambridge Problems," the solution of this problem is quite wrong.

A few years since, a problem was circulated among the teachers in this country, in which it was required to find the longest pole that can be run up a chimney, in which the height of the crossbar and the depth of the flue are given. This is obviously the same as the preceding.

In No. 4 of the Mathematical Miscellany, problem 7 is this: Required to find the locus of all the points so situated within a right angle, that the shortest line which can be made to pass through

each of them, and terminate in the sides of the right angle, shall be of a constant length. The equation of the required locus is readily found to be the same as (1).

This is also the equation of the locus of the consecutive intersections of a straight line of a given length, terminating in the sides of a right angle. If we take the origin at the right angle, then will

$$y = ax + b \tag{2}$$

be the equation of the line of given length; whence we shall have, since for $y = 0$, $x = -\frac{b}{a}$, and for $x = 0$, $y = b$

$$b^2 + \frac{b^2}{a^2} = r^2,$$

in which r is the given length.

With this eliminate a from (2), then will

$$y = \frac{bx}{(r^2 - b^2)^{\frac{1}{2}}} + b. \tag{3}$$

By differentiating (3) on the supposition that b only is variable, we get

$$b = r\left(1 - \left(\frac{x}{r}\right)^{\frac{2}{3}}\right)^{\frac{1}{2}},$$

and this value of b substituted in (3) gives

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = r^{\frac{2}{3}}.$$

If, in the common equation of the Hypocycloid, we put $r' = \frac{1}{4} r$, we get

$$x = r \sin^3 \theta, \qquad y = r \cos^3 \theta;$$

and from these equations we obtain

$$\int (dx^2 + dy^2)^{\frac{1}{2}} = 3 r \int \sin \theta \cos \theta \, d\theta = \tfrac{3}{2} r \sin^2 \theta = \tfrac{3}{2} r,$$

for the length of the curve between the limits $\theta = 0$, and $\theta = 90°$. Also, for the area between the same limits we find

$$\int y \, dx = 3 r^2 \int \sin^2 \theta \cos^4 \theta \, d\theta = \frac{3 \pi r^2}{32};$$

for the surface of the solid of revolution around the axis of x

$$\int 2 \pi y \, ds = 6 \pi r^2 \int \sin \theta \cos^4 \theta \, d\theta = -\frac{6 \pi r^2 \cos^5 \theta}{5} = \frac{6 \pi r^2}{5};$$

for the solidity of this solid of revolution,

$$\int \pi y^2 \, dx = 3 \pi r^3 \int \sin^2 \theta \cos^7 \theta \, d\theta = \frac{16 \pi r^3}{105}.$$

NOTE ON THE EQUATION OF THE PROJECTION OF A GREAT CIRCLE OF THE GLOBE ON A MERCATOR CHART.

BY PROF. W. CHAUVENET

LET the great circle make the angle α with the equator, and let its intersection with the equator be taken as the origin of rectangular coördinates. These coördinates on the chart are

$x =$ the longitude of a point of the projected curve, reckoned from the origin,

$y =$ the expanded arc of the meridian from the equator to the point.

If then φ is the latitude of the point (x, y), the radius of the sphere being unity, we have on the chart,

$$y = \text{Nap. log tan } \left(\frac{\pi}{4} + \tfrac{1}{2}\,\varphi\right),$$

and on the sphere,

$$\tan \varphi = \tan \alpha \sin x,$$

from which the equation of the projection is obtained as follows: If $e =$ the base of Naperian logarithms,

$$\tan \left(\frac{\pi}{4} + \tfrac{1}{2}\,\varphi\right) = e^{y}$$

$$\tan \left(\frac{\pi}{4} - \tfrac{1}{2}\,\varphi\right) = e^{-y},$$

the half difference of which is

$$\tan \varphi = \tfrac{1}{2}\,(e^{y} - e^{-y}) = \text{potential sine of } y,$$
$$= \text{Sin } y.$$

The required equation is, therefore,

$$\text{Sin } y = \tan \alpha \sin x.$$

The construction of the curve by points will then be easily effected with the aid of a table of potential functions.

NOTES ON THE THEORY OF PROBABILITIES.

By Simon Newcomb. Naut. Alm. Office, Cambridge, Mass.

1. The theory of probabilities, or doctrine of chances, as it has sometimes been called, being one of those subjects on which little has yet been written among us, it is proposed to develop its elementary principles through the pages of this Journal, and to apply them to a number of such problems as are suggested either by the observation of the material world, or by the affairs of society and of daily life.

2. Against the logical accuracy of the results of this calculus many objections have been raised. In the latter part of the last century, the Rev. Mr. Mitchell computed, that if the stars visible to the naked eye, had been scattered fortuitously over the heavens, there were five hundred thousand chances to one against any six of them falling together in so small a space as that occupied by the Pleiades. The correctness of this conclusion has lately been disputed by an eminent English scientist, who thinks that in the case supposed, such groupings of the stars are precisely what ought to be expected. This problem will be examined in the course of these notes as a particular case of one more general, and we shall find by a mode of reasoning entirely different from that of Mr. Mitchell, that he is substantially correct.

3. The object of the theory of probabilities is to determine the amount of reason which we have to expect any event when we have not the data to determine with certainty whether it will occur or not; and when the data of which we are possessed are such as to admit of the application of mathematical analysis. This amount of reason is not any quality of the event itself, but only a degree of rational expectation in the mind of the individual.

It is considered a well-established philosophical principle that the same causes will always produce the same effects; so that an intellect having a perfect knowledge of the former, and possessed of sufficient reasoning power, could always predict the latter. But in the external world, we can never arrive at any knowledge of the philosophical efficient cause, but can only observe that certain events have always been followed by certain other events. The preceding event is, then, generally denominated the cause, and the event which follows it the effect. Using the term cause in this sense, it generally happens, that so much of the cause as we are cognizant of may be followed by a great number of effects; and then we can only determine of the latter that they are more or less probable.

4. Hereafter, the operation of so much of the cause as we are cognizant of will be called a *trial;* and the effects of which we wish to find the probabilities will be designated as *events.* Generally the number of separate possible results of the trial will be greater than the number of events which we consider. The former will then be designated as *cases;* such of the cases as necessarily involve or produce the event in question are called *favorable,* while those which necessarily negate or prevent it are termed *unfavorable.* If the separate cases are all equally probable, the probability of the event is represented by the ratio of the number of the favorable cases to the whole possible number of cases.

5. To illustrate the above principles and definitions: suppose that a cube has the sides numbered from one to six; that sides one and two are white, and the remaining four black. Suppose, also, that the cube is thrown from a box at random, and the probability that a white side will fall upward is required. Then —

The given cause or *trial* is the throwing of the cube.

The *event* is the falling upward of a white side.

The number of possible *cases* is six; namely, the falling up of sides 1, 2, 3, &c., to 6 respectively.

The *favorable* cases are two, the falling of side 1, and of side 2.

Wherefore the *probability* of the event is $\frac{2}{6} = \frac{1}{3}$.

6. In general, if there are m cases equally probable, of which one must, and only one can, occur; and if p of these cases are favorable to the production of an event, the probability of that event is $\frac{p}{m}$. We see that if the probability is $\frac{1}{2}$, the event is just as likely to occur as to fail; and that if the probability is 1, the event becomes certain. All degrees of probability must be included between the limits 0 and 1; 0 being the symbol of impossibility and 1 of certainty.

7. In enumerating the different possible cases, we must take care so to subdivide them that they shall all be equally probable. Through not taking this precaution, an eminent mathematician of the last century fell into an error with regard to a very simple problem, of the following nature: If a coin is thrown twice, what is the probability that a head will fall upward? D'Alembert reasoned as follows: If head is thrown the first time, then the event occurs, and the throw need not be repeated; only three cases are therefore to be considered —

Heads the first time,
Tails the first time and heads the second,
Tails both times.

Two of these cases being favorable, the probability of the event is $\frac{2}{3}$. This result, however, is incorrect, because, as is easily seen, the probability of the first case alone is $\frac{1}{2}$, and therefore twice as great as that of either of the others. It ought, therefore, to be considered as subdivided into two separate cases, making the probability of the event $\frac{3}{4}$, which is the correct answer.

8. The following theorem will frequently save us the trouble of enumerating all the possible cases.

The probability that both of two independent events will happen, is equal to the product of their individual probabilities.

For the first event, let there be m cases, of which p are favorable, and for the second, n cases, of which q are favorable. If we give both events a trial, each of the m cases may be combined with any one of the n cases, making in all $m \times n$ possible combinations, each of which will be equally probable. The combinations favorable to *both* events will be those only in which one of the p cases, favorable to the first, is combined with one of the q cases, favorable to the second, the number of which will be $p \times q$. The probability of the compound event is therefore $\frac{p\,q}{m\,n}$, which is equal to the product of the separate probabilities.

As an example, suppose that with the cube spoken of in § 5 is put an octahedron, of which three sides only are white. If the two solids are thrown down at random, each of the six sides of the cube may be combined with any one of the eight sides of the octahedron, making the possible combinations (1, 1), (1, 2), (1, 3), &c. to (1, 8); (2, 1), (2, 2), (2, 3), &c. to (2, 8); .... (6, 1), (6, 2), &c. to (6, 8); in all, 48 combinations. If we determine the result of each of these combinations, we shall find that the upper side of

The cube will be white and the octahedron white in 6 cases.
" " " " white " " " black " 10 "
" " " " black " " " white " 12 "
" " " " black " " " black " 20 "

Dividing each of these results by 48, the whole number of cases, we shall find the probabilities of the several compound events to be equal to the product of the probabilities of the simple events which compose them; that is, $\frac{6}{48} = \frac{2}{6} \times \frac{3}{8}$; $\frac{10}{48} = \frac{2}{6} \times \frac{5}{8}$; $\frac{12}{48} = \frac{4}{6} \times \frac{3}{8}$; $\frac{20}{48} = \frac{4}{6} \times \frac{5}{8}$.

THE

MOTIONS OF FLUIDS AND SOLIDS

RELATIVE TO

THE EARTH'S SURFACE.

INTRODUCTION.

Some of the results contained in the following pages were published about two years ago, in an essay in the Nashville Journal of Medicine and Surgery, edited by Professor W. K. Bowling, M. D., of Nashville, Tennessee. A small edition was also published in pamphlet form, and distributed by the Smithsonian Institution and myself amongst various scientific men, libraries, and scientific associations, both in this country and in Europe. In that essay it was attempted to show that the depression of the atmosphere at the poles and the equator, and the accumulation or bulging at the tropics, as indicated by barometric pressure, the gyratory motion of storms from right to left in the northern hemisphere, and the contrary way in the southern, and certain motions of oceanic currents, are necessary consequences of the modifying forces arising from the earth's rotation on its axis, and also that the observed flowing of the lower strata of the atmosphere in the middle latitudes towards the poles, contrary to the ordinary theory of the trade winds, is caused by the greater pressure of this accumulation of atmosphere at the tropics. It is believed that that essay was the first attempt to account for those remarkable phenomena by means of

the modifying influence of the earth's rotation, and that it furnishes the only satisfactory explanation of them which has yet been given.

In that essay it was inconvenient to use any mathematical formulæ, and consequently the results merely of only a partial and imperfect investigation of the subject were given; but it is thought that on account of the importance of the subject, it deserves a more thorough investigation. It is proposed, therefore, in the following pages, to go into a complete analytical investigation of the general motions of fluids surrounding the earth, and of projectiles at its surface, arising from disturbing forces and the earth's attraction, combined with the modifying forces arising from its rotation on its axis. We shall accordingly, in the first section, investigate the general equations of motion relative to the earth's surface, applicable to both fluids and solids, and in the subsequent sections treat, first of the motions and figure of the whole or a part of a fluid surrounding the earth, upon the hypothesis that its motions are not resisted by the earth's surface, and then apply the results thus obtained to the explanation of the general motions of the atmosphere, the motions of storms or hurricanes, and the currents of the ocean. We shall also give a complete but concise treatise on projectiles, taking into account the effect of the earth's rotation.

We hope to be able in this investigation to give a satisfactory explanation of all the general motions of the atmosphere and of the ocean; the cause of the greater pressure of the atmosphere near the tropics than at the equator and the poles, and of the greater pressure generally in the northern hemisphere than in the southern; to account for the motion of all great storms in both hemispheres from the equator towards the poles in parabolic paths, and to completely establish their gyratory character; none of which phenomena have ever been satisfactorily accounted for by any of the usual theories, which do not take into account the influence of the earth's rotation.

SECTION I.

OF THE GENERAL EQUATIONS OF MOTION RELATIVE TO THE EARTH'S SURFACE.

1. Let x, y, and z be three rectangular coördinates, having their origin at the centre of the earth, x corresponding with the axis of rotation. Also let

Ω be the potential of all the attractive forces of the earth,
P the pressure of the fluid, and
k its density.

Then $k\,D_x\,\Omega$, $k\,D_y\,\Omega$, and $k\,D_z\,\Omega$ are the forces for a unit of volume, arising from the earth's attraction, and $D_x P$, $D_y P$, and $D_z P$, those arising from the pressure of the fluid, in the directions respectively of x, y, and z; and we have for the equations of the absolute motions of the fluid, regarding the centre of the earth at rest,

$$\text{(1)} \qquad \begin{aligned} D_t^2 x + D_x \Omega + \tfrac{1}{k} D_x P &= 0, \\ D_t^2 y + D_y \Omega + \tfrac{1}{k} D_y P &= 0, \\ D_t^2 z + D_z \Omega + \tfrac{1}{k} D_z P &= 0. \end{aligned}$$

Putting $P = 0$, they are the equations of a projectile.

2. Let r be the distance from the earth's centre,

θ, the polar distance,
φ, the longitude, and
n, the angular velocity of the earth's rotation.

Then we have

$$\text{(2)} \qquad \begin{aligned} x &= r\cos\theta, \\ y &= r\sin\theta\cos(nt+\varphi) = r\sin\theta\cos\omega, \\ z &= r\sin\theta\sin(nt+\varphi) = r\sin\theta\sin\omega, \end{aligned}$$

by putting for brevity $nt + \varphi = \omega$.

The position of the ordinates y and z, and also the origin of the

time t, being entirely arbitrary, they must be so taken as to make $\sin(nt + \varphi)$ vanish in the plane of x, y.

Using these values of x, y, and z in equations (1), we obtain equations in which the first derivatives of r, θ, and φ represent the motions of the fluid or projectile relative to the earth's surface.

3. Taking the first derivatives of (2) with regard to t, we get

$$D_t x = \cos\theta\, D_t r - r \sin\theta\, D_t \theta,$$
$$D_t y = \sin\theta \cos\omega\, D_t r + r \cos\theta \cos\omega\, D_t\theta - r \sin\theta \sin\omega\, D_t\omega,$$
$$D_t z = \sin\theta \sin\omega\, D_t r + r \cos\theta \sin\omega\, D_t\theta + r \sin\theta \cos\omega\, D_t\omega,$$

Taking the second derivatives, we get

$$D_t^2 x = \cos\theta\, D_t^2 r - 2\sin\theta\, D_t r\, D_t\theta - r\cos\theta\,(D_t\theta)^2 - r\sin\theta\, D_t^2\theta,$$

$$\begin{aligned} D_t^2 y = {} & \sin\theta\cos\omega\, D_t^2 r + 2\cos\theta\cos\omega\, D_t r\, D_t\theta - 2\sin\theta\sin\omega\, D_t\omega\, D_t r \\ & + r\cos\theta\cos\omega\, D_t^2\theta - r\sin\theta\cos\omega\,(D_t\theta)^2 - 2r\cos\theta\sin\omega\, D_t\omega\, D_t\theta \\ & - r\sin\theta\sin\omega\, D_t^2\varphi - r\sin\theta\cos\omega\,(D_t\omega)^2. \end{aligned} \tag{3}$$

$$\begin{aligned} D_t^2 z = {} & \sin\theta\sin\omega\, D_t^2 r + 2\cos\theta\sin\omega\, D_t r\, D_t\theta + 2\sin\theta\cos\omega\, D_t\omega\, D_t r \\ & + r\cos\theta\sin\omega\, D_t^2\theta - r\sin\theta\sin\omega\,(D_t\theta)^2 + 2r\cos\theta\cos\omega\, D_t\omega\, D_t\theta \\ & + r\sin\theta\cos\omega\, D_t^2\varphi - r\sin\theta\sin\omega\,(D_t\omega)^2. \end{aligned}$$

Since x, y, and z are functions of r, θ, and φ, we must put

$$\begin{aligned} D_x\Omega &= D_r\Omega \,.\, D_x r + D_\theta\Omega \,.\, D_x\theta + D_\phi\Omega \,.\, D_x\varphi, \\ D_y\Omega &= D_r\Omega \,.\, D_y r + D_\theta\Omega \,.\, D_y\theta + D_\phi\Omega \,.\, D_y\varphi, \\ D_z\Omega &= D_r\Omega \,.\, D_z r + D_\theta\Omega \,.\, D_z\theta + D_\phi\Omega \,.\, D_z\varphi. \end{aligned} \tag{4}$$

Now we have

$$r^2 = x^2 + y^2 + z^2,$$
$$\tan\theta = \frac{\sqrt{y^2+z^2}}{x},$$
$$\tan\omega = \frac{z}{y}.$$

Hence,

$$D_x r = \frac{x}{r} = \cos\theta,$$
$$D_y r = \frac{y}{r} = \sin\theta\cos\omega,$$
$$D_z r = \frac{z}{r} = \sin\theta\sin\omega,$$

$$D_x\theta = -\frac{\sqrt{y^2+z^2}}{r^2} = -\frac{\sin\theta}{r},$$

$$D_y\theta = \frac{xy}{r^2\sqrt{y^2+z^2}} = \frac{\cos\theta\cos\omega}{r},$$

$$D_z\theta = \frac{xz}{r^2\sqrt{y^2+z^2}} = \frac{\cos\theta\sin\omega}{r},$$

$$D_x\varphi = 0,$$

$$D_y\varphi = \frac{-z}{y^2+z^2} = \frac{\sin\omega}{r\sin\theta},$$

$$D_z\varphi = \frac{y}{y^2+z^2} = \frac{\cos\omega}{r\sin\theta}.$$

By means of these equations, equations (4) become

$$D_x\Omega = D_r\Omega\cos\theta - D_\theta\Omega\frac{\sin\theta}{r},$$

$$(5)\quad D_y\Omega = D_r\Omega\sin\theta\cos\omega + D_\theta\Omega\frac{\cos\theta\cos\omega}{r} - D_\varphi\Omega\frac{\sin\omega}{r\sin\theta}.$$

$$D_z\Omega = D_r\Omega\sin\theta\sin\omega + D_\theta\Omega\frac{\cos\theta\sin\omega}{r} + D_\varphi\Omega\frac{\cos\omega}{r\sin\theta}.$$

In the same manner we obtain

$$D_xP = D_rP\cos\theta - D_\theta P\frac{\sin\theta}{r},$$

$$(6)\quad D_yP = D_rP\sin\theta\cos\omega + D_\theta P\frac{\cos\theta\cos\omega}{r} - D_\varphi P\frac{\sin\omega}{r\sin\theta},$$

$$D_zP = D_rP\sin\theta\sin\omega + D_\theta P\frac{\cos\theta\sin\omega}{r} + D_\varphi P\frac{\cos\omega}{r\sin\theta}.$$

Substituting the values of the first members of equations (3), (5), and (6) in equations (1), and multiplying them respectively by $\cos\theta$, $\sin\theta\cos\omega$, $\sin\theta\sin\omega$, and adding, we obtain the first of the following equations. Again, multiplying them respectively by $r\sin\theta$, $-r\cos\theta\cos\omega$, and $-r\cos\theta\sin\omega$, and adding, we obtain the second of those equations. Finally, multiplying the last two respectively by $r\sin\theta\sin\omega$, and $-r\sin\theta\cos\omega$, and adding, we get the last of the following equations:—

$$\frac{1}{k}D_rP = -D_t^2r + r(D_t\theta)^2 + r\sin^2\theta(n + D_t\omega)D_t\varphi + rn^2\sin^2\theta - D_r\Omega,$$

$$(7)\quad \begin{aligned}\frac{1}{k} D_\theta P &= -r^2 D_t^2 \theta - 2r D_t r D_t \theta + r^2 \sin\theta\cos\theta\,(n + D_t\omega) D_t\varphi \\ &\quad + r^2 n^2 \sin\theta\cos\theta - D_\theta \Omega, \\ \frac{1}{k} D_\phi P &= -r^2 \sin^2\theta D_t^2 \varphi - 2r \sin^2\theta D_t\omega D_t r \\ &\quad - 2r^2 \sin\theta\cos\theta D_t\omega D_t\theta - D_\phi \Omega.\end{aligned}$$

In these equations $D_r P$, $\frac{1}{r} D_\theta P$, and $\frac{1}{r\sin\theta} D_\phi P$ represent the forces arising from the pressure in the directions respectively of r, θ, and φ.

4. If N be the normal distance to the surface of the earth, or to any level surface, and the forces in the first two of the preceding equations be resolved in the directions of the normal and a perpendicular to the normal in the plane of the meridian, putting $\cos\frac{r}{N} = 1$, and neglecting the small terms multiplied by $\sin\frac{r}{N}$, which are of the second order of the earth's ellipticity, and letting $\frac{1}{r} D_{\theta'} P$ represent the force arising from the pressure, resolved in the direction of the perpendicular to the normal in the plane of the meridian, the preceding equations give, when the fluid is at rest,

$$(8)\quad \begin{aligned}\frac{1}{k} D_N P &= r n^2 \sin^2\theta - D_N \Omega = -g, \\ \frac{1}{k} D_{\theta'} P &= r^2 n^2 \sin\theta\cos\theta - D_{\theta'} \Omega = 0, \\ \frac{1}{k} D_\phi P &= - D_\phi \Omega = 0,\end{aligned}$$

and hence, neglecting the very small terms multiplied by $\sin\frac{r}{N}$, depending upon the motions of the fluid relative to the earth's surface, they give for the fluid in motion,

$$(9)\quad \begin{aligned}\frac{1}{k} D_N P &= - D_t^2 r + r\,(D_t\theta)^2 + r\sin^2\theta\,(n + D_t\omega) D_t\varphi - g, \\ \frac{1}{k} D_{\theta'} P &= -r^2 D_t^2 \theta - 2r D_t r D_t\theta + r^2 \sin\theta\cos\theta\,(n + D_t\omega) D_t\varphi, \\ \frac{1}{k} D_\phi P &= -r^2 \sin^2\theta D_t^2 \varphi - 2r \sin^2\theta D_t\omega D_t r \\ &\quad - 2r^2 \sin\theta\cos\theta D_t\omega D_t\theta.\end{aligned}$$

Integrating, we obtain

$$\begin{aligned} P &= H - \int_N g\,k = H - g\,k\,N + \int_N N\,D_N(g\,k), \\ &= H - g\,k\,N + \int_N g\,N\,D_N k + \int_N N k\,D_N g, \\ &= H + K, \end{aligned} \tag{10}$$

in which H must satisfy the following equations of partial differentials: —

$$\begin{aligned} \frac{1}{k} D_N H &= -D_t^2 r + r\,(D_t\vartheta)^2 + r\sin^2\vartheta\,(n + D_t\omega)\,D_t\varphi, \\ \frac{1}{k} D_{\vartheta'} H &= -r^2 D_t^2\vartheta - 2\,r\,D_t r\,D_t\vartheta + r^2\sin\vartheta\cos\vartheta\,(n + D_t\omega)\,D_t\varphi, \\ \frac{1}{k} D_\phi H &= -r^2\sin^2\vartheta\,D_t^2\varphi - 2\,r\sin^2\vartheta\,D_t\omega\,D_t r \\ &\quad - 2\,r^2\sin\vartheta\cos\vartheta\,D_t\omega\,D_t\vartheta, \end{aligned} \tag{11}$$

and in which

$$K = -g\,k\,N + \int_N g\,N\,D_N k + \int_N N k\,D_N g. \tag{12}$$

Hence H is the pressure arising from the motions of the fluid, and K that arising from its gravity. If g and k are functions of N, ϑ', and φ, K is a function of the same.

5. For a stratum of equal pressure, P is constant, and hence

$$D_{\vartheta'} P = 0, \qquad D_\phi P = 0.$$

If we therefore put K' and h for the special values respectively of K and N, belonging to a stratum of equal pressure, and take the derivatives of (10) with regard to ϑ' and φ, and neglect the very small terms containing $D_t r$ as a factor, which, in all ordinary motions of the fluid, will be shown to be insensible, we obtain for the general equations of horizontal motions, by restoring the value of ω in § (2).

$$\begin{aligned} 0 &= \frac{1}{k} D_{\vartheta'} K' - r^2 D_t^2\vartheta + r^2\sin\vartheta\cos\vartheta\,(2n + D_t\varphi)\,D_t\varphi, \\ 0 &= \frac{1}{k} D_\phi K' - r^2\sin^2\vartheta\,D_t^2\varphi - 2\,r^2\sin\vartheta\cos\vartheta\,(n + D_t\varphi)\,D_t\vartheta, \end{aligned} \tag{13}$$

in which

$$K' = -g\,k\,h + \int_h g\,h\,D_h k + \int_h h\,k\,D_h g. \tag{14}$$

6. If we suppose the fluid to be elastic, and the ratio of the density to the elastic force or pressure to depend upon the temperature, we may put

$$k = \alpha P, \tag{15}$$

in which α may be a function of h, θ', and φ. Substituting this value of k in (14), we get, when g may be regarded as constant, since in that case the last term of (14) vanishes.

$$\begin{aligned} D_{\theta'} K' &= - D_{\theta'} (g \alpha P h) + D_{\theta'} \int_h g h D_h (\alpha P), \\ &= - g \alpha P D_{\theta'} h - g h P D_{\theta'} \alpha + g P D_{\theta'} \int_h h D_h \alpha, \end{aligned} \tag{16}$$

Hence, dividing by $k = \alpha P$, we get

$$\begin{aligned} \frac{1}{k} D_{\theta'} K' &= - g D_{\theta'} h - g h D_{\theta'} \log \alpha + A_{\theta'}, \\ \frac{1}{k} D_{\varphi} K' &= - g D_{\varphi} h - g h D_{\varphi} \log \alpha + A_{\varphi}, \end{aligned} \tag{17}$$

by changing θ' to φ for the last equation, and putting

$$\begin{aligned} A_{\theta'} &= \frac{g}{\alpha} D_{\theta'} \int_h h D_h \alpha, \\ A_{\varphi} &= \frac{g}{\alpha} D_{\varphi} \int_h h D_h \alpha. \end{aligned} \tag{18}$$

7. When $D_h \alpha$ is constant, that is, when α varies as the altitude,

$$\begin{aligned} A_{\theta'} &= \frac{g e}{2 \alpha} D_{\theta'} h^2, \\ A_{\varphi} &= \frac{g e}{2 \alpha} D_{\varphi} h^2, \end{aligned} \tag{19}$$

in which

$$e = D_h \alpha,$$

depending upon the constant ratio of increase or decrease of α with h.

8. When α is constant, or when the fluid is homogeneous, the last two terms of (17) vanish, and in the latter case, h may represent the height of the surface of the fluid above any level surface.

9. In the preceding investigation, the effect upon g arising from a change of the figure of the fluid has been neglected. It is small

in the case of water, and in the case of the atmosphere, entirely insensible.

10. Equations (13), together with the condition that the same amount of fluid must always occupy a space which is inversely as its density, the analytical expression of which is called the equation of continuity, are the conditions which must be satisfied by the motions of a fluid surrounding the earth, and are sufficient to determine its horizontal motions, and also the value of h, which gives the figure of the fluid. When h is determined, equation (10) gives the pressure of the fluid. As only a very special form of the general equation of continuity will be needed in this investigation, it is unnecessary to give it here.

[TO BE CONTINUED.]

PROBLEM.

BY J. FOSTER FLAGG, C. E.

REQUIRED a formula for the strength of a circular flat iron plate of uniform thickness, supported throughout its circumference and uniformly loaded; likewise a formula, supposing the plate to be bolted down. Also find the equation of the curve, for each case, which should be given to a section of the plate, in order that it may have the greatest strength with the least material. The problem may be further varied by supposing the plate *square*, instead of round.

We have recently had great need of such formulas on the Washington Aqueduct, and if they were obtained in a moderately simple shape, they would, I think, be quite valuable. I know of no place where the discussion of these questions may be found, although it is quite possible that it may have been made.

Mathematical Monthly Notices.

I. *Examples of the Processes of the Differential and Integral Calculus.* Collected by D. F. Gregory, M. A., late Fellow of Trinity College. Second Edition, edited by William Walton, M. A., Trinity College.

II. *Problems in Illustration of the Principles of Plane Coördinate Geometry.*

III. *A Collection of Problems in Illustration of the Principles of Theoretical Mechanics.* Second Edition, with numerous Alterations and Additions.

IV. *A Collection of Problems in Illustration of the Principles of Theoretical Hydrostatics and Hydrodynamics.*

V. *A Collection of Problems in Illustration of the Principles of Elementary Mechanics.* By William Walton, M. A., Trinity College, Cambridge; Mathematical Lecturer at Magdalene College. Cambridge: Deighton, Bell & Co.

We include these works prepared by Mr. Walton in the same notice, because their aim and spirit are the same; and any remarks applicable to one will apply with more or less force to all. The simple titles give a good idea of the place they are intended to fill, and it only remains to call attention to the plan upon which they are constructed.

In the Preface to II., Mr. Walton says: "The tendency of even detached and desultory problems is no doubt to sharpen the intellect of the learner, and to give him a true conception of the signification of corresponding and more abstract propositions; the classification, however, of problems by some law of affinity is, in my opinion, calculated not only to produce the same result in a still higher degree, but also to qualify him by an enlarged comparison for becoming skilful and judicious in effecting his own solutions. In this conviction has originated the composition of the present work."

This quotation clearly shows the one governing principle which the author has kept steadily in view in their preparation. We have long felt, that, especially for the student who wishes such a thorough knowledge of the higher mathematics as will enable him to use them successfully as an instrument, something more was needed than the simple development of theory, with applications to a few special problems intended to elucidate it, rather than show its power as a means of investigation; that collections of problems were needed in the various branches of pure and applied mathematics, so graded with respect to difficulty as always to tax his ability yet never to discourage his efforts, and so classified with regard to the principles involved, that he may always be sure of his knowledge, though he may often want the skill to solve them; that such aid should be given in the solution of problems of special difficulty, with such general suggestions upon all, as would stimulate effort and not preclude its necessity; that such problems should be especially noticed as have an historical value, and the solutions of which have been instrumental in inventing or perfecting theory; that such references should be made to the original sources of information respecting the authorship and full discussion of typical or characteristic problems, as would enable the student to consult them for himself. In the above volumes these wants are, to a good degree, supplied.

For instance, Volume II. is arranged under the following heads; each head being divided into sections which are severally devoted to a special property. Straight Line, in 7 sections, including 143 problems; Circle, in 12 sections, with 93 problems; Parabola, in

20 sections, with 128 problems; ELLIPSE, in 27 sections, with 202 problems; HYPERBOLA, in 20 sections, with 135 problems; LINES OF THE SECOND ORDER, in 28 sections, with 171 problems.

Besides this large number of interesting problems, there are many elegant solutions, with over five hundred references to all the Mathematical Journals, as well as authors, from Apollonius down to the present time. A like analysis would show the other volumes to be, in all important particulars, equally valuable. They are all intended as supplementary to any of the text-books upon the respective subjects, and to be used in connection with them.

The science of mathematics, and the *science of teaching* mathematics, are very different; and while Mr. WALTON may not claim to have added much to the former science by preparing these volumes, he may fairly claim that he has at least fully developed an important and fundamental principle in any thorough system of mathematical instruction, and contributed, in no small degree, towards elevating the profession of teaching to the dignity of a science. Instead of simply studying mathematics as an end to themselves, or as the means to a more material end, he has also studied them in their relations to the laws which govern mental growth, and as a means of mental culture and development. If this brief notice shall bring these volumes to the attention of that class of students who can appreciate and profit by them, our object in noticing them will be accomplished.

The Lady's and Gentlemen's Diary; or, Poetical and Mathematical Almanac for the year 1859. London: Printed for the Company of Stationers.

This is the 156th number of this celebrated Annual. Besides the solutions of the fifteen problems proposed in the number for 1858, we find fifteen new ones, the solutions of which will be published in the next number. We quote No. XV., hoping that some of our readers may be induced to compete for the prize offered for its solution.

PRIZE QUESTION; by Mr. C. H. BROOKS, C. E., Newcastle-upon-Tyne.

"Two rods, two and four feet long respectively, having their middle points connected by a string one foot in length, are thrown up; show that the chance of their crossing is $\frac{1}{2} + \frac{2}{\pi^2}$."

The number closes with interesting "Mathematical Papers" by STEPHEN FENWICK, F. R. A. S., of the Royal Military Academy, Woolwich; T. T. WILKINSON, F. R. A. S.; W. S. B. WOOLHOUSE, London; and Mr. SAMUEL BILLS, of Hawton, near Newark-upon-Trent. To some of these papers we shall have occasion to refer more particularly hereafter.

Editorial Items.

Solutions of the Prize Problems in the October Number of the Monthly.

THE solutions which we have the pleasure of announcing were, much to our regret, placed in the hands of the Committee at too late a date to allow sufficient time to report in this number of the Monthly. The report of the Committee, with the Prize Solutions, will be published in the next number. We cannot, however, omit this opportunity to express our satisfaction, with the unmistakable evidence before us, of the good influence our prizes are exerting; and we feel sure, that no one who has sent us solutions will regret the labor he has expended upon them, whether successful in securing a prize or not. It must not be forgotten, however,

that the best solution of each problem is published as the "prize solution," thus making it desirable for students to send even a single solution. We are much obliged to those who have sent us solutions, marked "not for a prize;" and shall be glad if all taking sufficient interest in the Prize Problems to solve them, will send us their solutions. Lastly, we beg those whose names are found in the following announcement to accept our sincere thanks for the interest they have taken in this department of the Monthly.

CLEVELAND ABBY, Student in Astronomy, University of Michigan, answered all the questions. (Dr. Brünnow, Prof.)

GEORGE A. OSBORNE, Jr., Student in the Lawrence Scientific School, answered all the questions. (H. L. Eustis, Prof.)

G. W. HILL, Class of 1859, Rutger's College, New Brunswick, N. J., answered all the questions. (Dr. Strong, Prof.)

Student in Brown University, Providence, R. I., answered all the questions. (Name of student not received.)

ARTHUR WILKINSON, Junior Class, Harvard College, answered all but Prob. V. (Benjamin Peirce, Prof.)

JOSEPH TURNBULL, Salineville, Columbiana Co., answered all but Prob. II.

E. P. AUSTIN, Student in the University of Michigan, answered all the questions. (John E. Clark, Prof.)

ASHER B. EVANS, Junior Class, Madison University, Hamilton, N. Y., answered all the questions. (L. M. Osborn, Prof.)

H. G. PALFREY, Junior Class, Harvard College, answered all the questions. (Benjamin Peirce, Prof.)

CHARLES W. HASSLER, Student in Columbian College, Washington, D. C., answered all the questions. (Edward T. Fristoe, Prof.)

GEORGE W. FISHER, Senior Class, Yale College, New Haven, Ct., answered all the questions. (H. A. Newton, Prof.)

FRANCIS E. TOWER, Junior Class, Amherst College, Amherst, Mass., answered all the questions. (E. S. Snell, Prof.)

H. F. FISK, Junior Class, Wesleyan University, Middletown, Ct., answered all but Prob. III. (J. M. Van Vleck, Prof.)

GEORGE B. HICKS, Student, Cleveland, Ohio, answered all the questions.

J. A. WILLIAMSON, Student in Normal College, North Carolina, answered all the questions. (Lemuel Johnson, Prof.)

WILLIAM EVERETT, Senior Class, Harvard College, answered all the questions. (Benjamin Peirce, Prof.)

WILLIAM C. CLEVELAND, Student in the Lawrence Scientific School, answered all the questions. (H. L. Eustis, Prof.)

WALLER HOLLADAY, Student of Mathematics in the University of Va., answered all the questions. (A. T. Bledsoe, Prof.)

THEODORE COOPER, Cooper's Plains, New York, answered all but Prob. II.

W. F. OSBORNE, Sophomore Class, Wesleyan University, answered all the questions. (J. M. Van Vleck, Prof.)

JOSEPH METZGER, Student in Commercial and Classical Institute, New York, answered Probs. II. III. IV. (John Livor, Principal.)

P. BARTON, Student, Amsterdam, New York, answered all but Prob. V.

W. R. MCCRARY, Student in the Military Institute, Franklin Co., Ky., answered all the questions. (R. J. Adcock, Prof.)

J. GIBSON CANAN, Student in Washington College, Chestertown, Md., answered all but Prob. V., and the first equation in Prob. I.

WILLIAM P. TEN BROECK, Student, La Fayette, Ind., answered Probs. I. and II.

GEORGE W. PIERCE, Student in the Classical and Mathematical School of Profs. Lovering and Lane, Cambridge, Mass., answered all the questions, but equation 3, Prob. I.

JOSEPH P. FRIZELL, Student in the office of J. B. Henck, Civil Engineer, Boston, answered all the questions but equation 3, Prob. I.

O. B. WHEELER, Student in the University of Michigan, Ann Arbor, answered all the questions. (D. Wood, Prof.)

F. W. BARDWELL, B. S., a graduate of the Lawrence Scientific School, Cambridge, Mass., and Assistant in the office of the American Ephemeris and Nautical Almanac, has accepted the appointment of Professor of Mathematics, Astronomy, and Civil Engineering in Antioch College, Yellow Springs, Ohio, and will enter upon the duties of the office Jan. 5, 1859. We are satisfied that the College has made a wise selection, and that Mr. BARDWELL will prove himself a thorough and faithful as well as able teacher. . . . . Authors will much oblige us if they will draw their figures just as they wish them engraved, and upon as small a scale as is consistent with clearness. For size, see cuts in the Monthly. . . . . All problems, especially those intended for insertion among the Prize Problems, should be accompanied with their solutions, to enable us readily to judge of their fitness for the purpose; otherwise their insertion must generally be delayed until we have time to examine them. We hope our friends will continue to send us such original problems as may from time to time occur to them. . . . . The desirableness of giving each author's profession and location was not suggested to us until some of the pages of our present number were electrotyped; which will account for the want of uniformity in this respect. . . . . In the demonstration of Prop. 7, page 84, there is a slight oversight. The constant C equals zero, instead of $\frac{1}{6}a^3$. The removal of this term from the equation will give the correct values of $x = \frac{(1 \pm \frac{1}{2}\sqrt{3})a}{8}$ . . . . . It gives us pleasure to add the following names to our list of coöperators and contributors: JOHN MASON BROWN, Esq., Frankfort, Ky.; GEORGE W. HILAND, Esq., Adams, Seneca Co., Ohio; Prof. O. H. LELAND, Baylor University, Independence, Texas; ARTEMAS MARTIN, Esq., Franklin, Venango Co., Penn.; GEORGE STUART, Esq., Tutor in Haverford College, West Haverford, Penn.; Captain D. P. WOODBURY, U. S. Corps of Engineers, Tortugas, Florida. . . . . BOOKS RECEIVED. — Papers on Practical Engineering, published by the Engineer Department, for the Use of the Officers of the United States Corps of Engineers. — No. 1. Bitumen; by Lieut. H. WAGER HALLECK, U. S. Corps of Engineers. (Out of Print). — No. 2. Sea-Wall, on Lovell's Island; by Brevet-Colonel S. THAYER, U. S. Corps of Engineers. Washington, 1844. — No. 3. Sustaining Walls; by Lieut. D. P. WOODBURY, U. S. Corps of Engineers. Washington: Taylor & Maury, 1854. — No. 4. Military Bridges; by Capt. GEORGE W. CULLUM, U. S. Corps of Engineers. New York: D. Appleton & Co., 1849. — No. 5. On the Resistance of Piles to Superincumbent Pressure; by Brevet Lieut.-Colonel JAMES L. MASON, Capt. U. S. Engineers. Washington, 1850. — No. 6. On the Effects of Firing with Heavy Ordnance from Casemate Embrasures, and also the Effects of Firing against the same Embrasures with various Kinds of Missiles; by Brevet Brigadier-General JOSEPH G. TOTTEN, Colonel and Chief Engineer U. S. Army. Washington: Taylor & Maury, 1857. — No. 7. Treatise on the various Elements of Stability in the Well-Proportioned Arch, with numerous Tables of the Ultimate and Actual Thrust; by Capt. D. P. WOODBURY, U. S. Corps of Engineers. New York: D. Van Nostrand, 1858. — Nouvelles Annales de Mathématiques, Novembre, 1858.

ANNOUNCEMENT.

We are at present able to announce, that the following Series of Papers and Treatises will appear in the Mathematical Monthly. Some of them are already in manuscript, others in course of preparation; and all of them will be commenced at the earliest practicable date.

I. A Series of Papers on the Application of the Doctrine of Probabilities to Vital Statistics in the Construction of Life and Population Tables, and in the Solution of important Monetary and other practical Problems, involving Life Contingencies. By E. B. Elliott, Esq.

II. On the Motions of Fluids and Projectiles relative to the Earth's Surface. By Prof. Ferrel.

III. A Series of Papers on the Theory of Least Squares, with Practical Applications. By J. E. Hilgard, Esq.

IV. A Treatise on the Elements of Plane Geometry. By Rev. Thomas Hill.

V. A Treatise on Curves of Single Curvature. By Rev. Thomas Hill.

VI. A Series of Notes on the Calculus of Probabilities. By Simon Newcomb, Esq.

VII. A Treatise on Determinants. By James Edward Oliver, Esq.

VIII. A Treatise on Analytic Morphology. By Prof. Benjamin Peirce.

IX. Researches in the Mathematical Theory of Music. By Truman Henry Safford, Esq.

X. A Treatise on One-Plane-Descriptive Geometry. By William Watson, Esq.

XI. A Treatise on the Calculus of Operations. By Prof. George C. Whitlock.

Besides the above, we have received, either in manuscript or by title, a large number of notes and papers upon nearly as large a variety of subjects; so large, indeed, that there is not the least doubt about our being able to execute the proposed plan of the Monthly in all its details. The only difficulty will be a want of space; but this we shall remedy by increasing the number of pages, just as fast as the subscription will warrant. To this end, we trust that all interested will take such measures as they think best to increase its circulation.

PHILOSOPHICAL
INSTRUMENTS AND APPARATUS,

MANUFACTURED AT 313 WASHINGTON STREET, BOSTON,

BY E. S. RITCHIE,

FOR ILLUSTRATING THE SCIENCES OF

Pneumatics, Electricity, Chemistry, Optics, Hydrostatics, Hydraulics, Steam, Magnetics, Acoustics, Mechanics, Astronomy, Meteorology, &c., with over 600 pieces; also, Mathematical and Engineering Instruments.

Each article is warranted perfect. In designing the form, simplicity, as well as combination of parts in the formation of new Instruments, is carefully studied; and to this end only two sizes of screw connections are used.

Ritchie's Illustrated Catalogue, which will be sent by mail on application, contains cuts of over 200 pieces, including his improved Air Pump, Electrical Machine, Ruhmkorff and Atwood Apparatus, &c., the University of Mississippi Electrical Machine having two six feet diameter plates, and four pairs of rubbers; also, sets made up to assist purchasers in selecting, at prices from $100 to $1,250 per set; and commendatory letters from eminent Physicists in various parts of the country who are using his apparatus.

WILLIAM BOND AND SON,

17 CONGRESS STREET, BOSTON,

Chronometer Makers to the United States Government,

HAVE FOR SALE

A Complete Assortment of Marine and Sidereal Chronometers,

ASTRONOMICAL CLOCKS FOR OBSERVATORIES,

WITH THE MOST APPROVED DESCRIPTION OF COMPENSATING PENDULUMS.

They also manufacture the SPRING GOVERNOR APPARATUS, for recording Astronomical Observations by the aid of Electro Magnetism, known as the American Method, and adopted at the Washington and Cambridge Observatories in the United States, and Greenwich and other Observatories in Europe.

Astronomical Instruments, Transits, Telescopes, &c.,

Of the usual description. Also, those for use at fixed Observatories, of larger dimensions, imported to order.